Direct Gear Design: Bending Stress Minimization

Alexander L. Kapelevich and Yuriy V. Shekhtman

Nomenclature

a_w	center distance
b	face width
C_{tt}	tooth thickness ratio
bl	operating backlash
d_a	outside circle diameter
d_r	root circle diameter
J	bending stress geometry factor
k_b	bending stress balance coefficient
K_f	stress concentration factor
ro	operating pitch diameter runout
S	bending stresses
S_w	operating pitch diameter tooth thickness
T	torque
α_w	operating pressure angle
δa_w	center distance variation
δ_S	balance tolerance
Δd_a	outside circle diameter tolerance
Δd_r	root circle diameter tolerance
ΔS_{wp}	tooth thickness tolerance

Subscript

1	pinion
2	gear

Introduction

Bending stress evaluation in modern gear design is generally based on the more-than-one-hundred-year-old Lewis equation. This equation, applied with the stress concentration factor K_f, defines the bending stress geometry factor J for traditionally designed standard or close-to-standard gears. The stress concentration factor K_f is defined from photoelastic experiments (Ref. 1) for rack-generated gears with standard tooth profile proportions.

The Direct Gear Design method (Ref. 2) is not constrained by a choice of gear tooth profiles based on standard tool parameters and uses nonstandard tooth shapes to provide required performance for a particular custom application. This makes finite element analysis (FEA) more preferable than the Lewis equation for bending stress definition. This paper does not describe the FEA application for comprehensive stress analysis of the gear teeth. It presents the engineering method for bending stress balance and minimization.

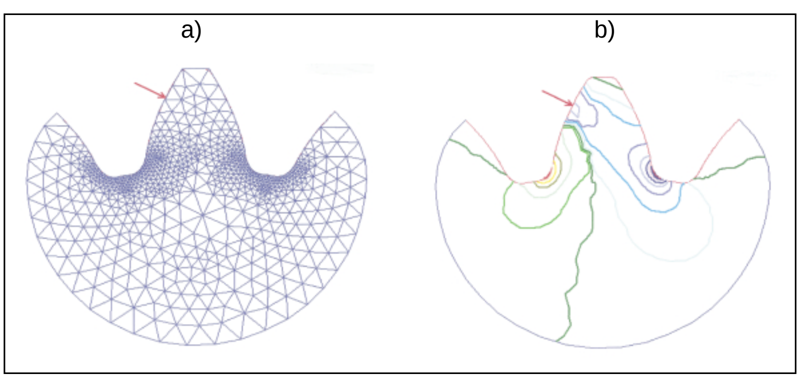

Fig. 1—The FEA gear tooth model: a) FEA mesh, b) stress isograms.

Tooth Modeling

The Direct Gear Design method defines parameters of the gear mesh to provide complete geometry of the involute profile of the teeth, including the base diameter, form diameter, outside diameter, tooth thickness, tip radii, etc. The fillet profile initially is defined as a trace of the tip of the mating gear tooth. This kind of fillet profile is used for plastic molded gears (Ref. 3).

The 2-D FEA model in Figure 1a presents a gear tooth profile that is limited from the sides and bottom by a constrained border with stationary nodes. All other nodes on the tooth profile and inside the tooth contour are movable. The fillet portion of the tooth profile (where maximum bending stress is expected) has equally spaced nodes with higher density (number of nodes per unit of profile length) than the rest of the tooth profile. The nodes on the involute profiles and the top land are located to have higher density close to the fillets and lower density in the top part of the tooth. The number of tooth profile nodes and the node density coefficient (ratio of the fillet profile node density to an average node density of the involute and top land profiles of the tooth) are selected. Fewer tooth profile nodes and lower node density coefficients yield less accurate stress calculations. Selection of larger numbers of tooth profile nodes and high node density coefficients provides a more accurate result, but increases calculation time. In most cases, 80–100 tooth profile nodes and node density coefficients of 1.75–2.5 were used.

The tooth load distribution problem is considered to define a value, a set of application point coordinates, and the direction of the force resulting in maximum bending stress. The friction effect at the contact point has been ignored. The load application point typically does not exactly match with a tooth profile node. It is replaced by a pair of forces that are applied to the two closest nodes above and below the load application point (Fig. 1a). The combined load value of those forces equals an initial load and distributed reversal proportional to the distances between the nodes and the load application point.

The automatically generated FEA mesh and bending stress isograms are shown in Figures 1a and 1b.

Bending Stress Balance

The pinion and the gear typically have different tooth shapes and face widths, and they could be made out of different materials or have different heat treatments, etc. In order to provide equally strong teeth of the pinion and the gear, their maximum bending stresses should be balanced. The balance condition providing equal bending safety factors for the pinion and the gear is

$$S_{\max1} - k_b \bullet S_{\max2} < \delta_s \qquad (1)$$

where

$S_{\max1}$ and $S_{\max2}$ are maximum bending stresses in the fillet area of the pinion and the gear,

k_b is the bending stress balance coefficient reflecting the difference of material properties (allowable stresses) and the number of tooth load cycles for the pinion and the gear, and

δ_s is the permissible balance tolerance (typically less than 1%).

In order to satisfy the condition of Equation 1, the bending stress balance FEA program changes the tooth thickness ratio

$$C_{tt} = S_{w1}/S_{w2} \qquad (2)$$

where

S_{w1} and S_{w2} are tooth thicknesses on operating pitch diameters (Fig. 2).

For gears that are designed using the traditional tool parameter approach, the tooth thicknesses S_{w1} and S_{w2} are changed by moving the tooling rack profile in or out of the center of the gears. Direct Gear Design changes the tooth thicknesses S_{w1} and S_{w2} while keeping certain conditions, such as the constant tooth top land thicknesses or the equal maximum specific sliding velocities for the pinion and the gear, etc. The fillet profiles, in this case, are still defined as traces of the tips of the mating gear teeth.

Sometimes the bending stress balance could be compromised to improve the performance parameters that have higher priority for particular gear applications (higher efficiency, lower noise, etc.).

Bending Stress Minimization

Bending stress minimization is the result of the definition of the fillet profile that provides minimum bending stress concentration and satisfies certain conditions (manufacturability, for example). There are different solutions to this problem. They are based on a curve fitting technique when the trochoid fillet profile, typical for the rack or mating gear generative method, is replaced by a parabola, ellipsis, chain line, or other curve reducing the bending stress. One of these solutions, using the fitted polynomial curve, is presented in U.S. Patent #6164880 (Ref. 4).

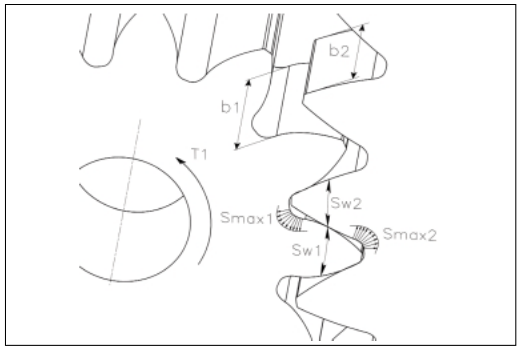

Fig. 2—Bending stress balancing.

This paper presents the fillet optimization that is based on three major components:

- random search method locating fillet points;
- trigonometric functions for fillet profile approximation;
- FEA for stress calculation.

An initial fillet profile is the trace of the mating gear tooth. This profile is the border limiting the optimization search area from the top to avoid interference with the mating gear. The first and the last fillet points lay on the form diameter circle and cannot be moved during an optimization process. The random search method is moving the fillet nodes (except first and last) along the beams that pass through the fillet center and the nodes of the initial fillet profile (see Fig. 3). The center of the fillet is the center of the best-fitted circle. The bending stresses are calculated for every new fillet point's combination. The program analyzes successful and unsuccessful steps, finding the direction of altering the fillet profile to reduce the maximum bending stress. The number of iteration steps (or optimization time) is limited. Extensive testing of this program allowed defining the set of random search parameters that provides satisfying solutions for all possible combinations of gear parameters. The random nature of this method does not repeatedly give absolutely identical results for the same set of gear parameters and number of iteration steps. The program was adjusted to provide the maximum bending stress difference for repeatable calculation not to

Dr. Alexander L. Kapelevich

is an owner of the consulting firm AKGears LLC, developer of modern Direct Gear Design[SM] methodology and software. He has over 20 years of experience in the development of aviation and commercial gear transmissions. He can be reached by e-mail at ak@akgears.com.

Dr. Yuriy V. Shekhtman

is an expert in mathematical modeling and stress analysis with 30 years of experience. A software developer for AKGears LLC, he created a number of computer programs based on FEA and other numerical methods. He can be reached by e-mail at ys@akgears.com.

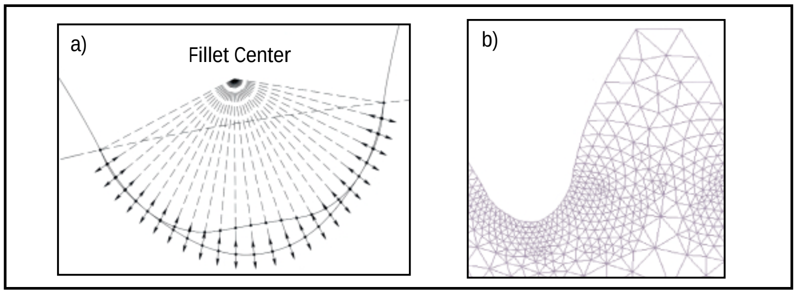

Fig. 3—Fillet profile optimization; a) random search node locating, b) FEA mesh around the optimized fillet.

Table 1—Fillet optimization of standard rack-generated gears.

Pressure angle, °		20		25		28	
Diametral Pitch, 1/in.		10		10		10	
Addendum, in.		0.1		0.1		0.09	
Whole depth, in.		0.22		0.22		0.198	
Tool tip radius, in.		0.03		0.03		0.0348	
Pinion Torque, in.-lb.		200		200		200	
		Pinion	Gear	Pinion	Gear	Pinion	Gear
Number of teeth		12	41	12	41	12	41
Tooth tip radius, in.		0.015	0.015	0.015	0.015	0.015	0.015
Face width, in.		0.5	0.5	0.5	0.5	0.5	0.5
Bending Stress (psi)	Standard	28,890	20,460	22,930	18,560	20,440	17,080
	Balanced	22,180	22,010	19,900	19,800	18,330	18,170
	Optimized	16,900	16,870	16,140	16,110	15,820	15,670

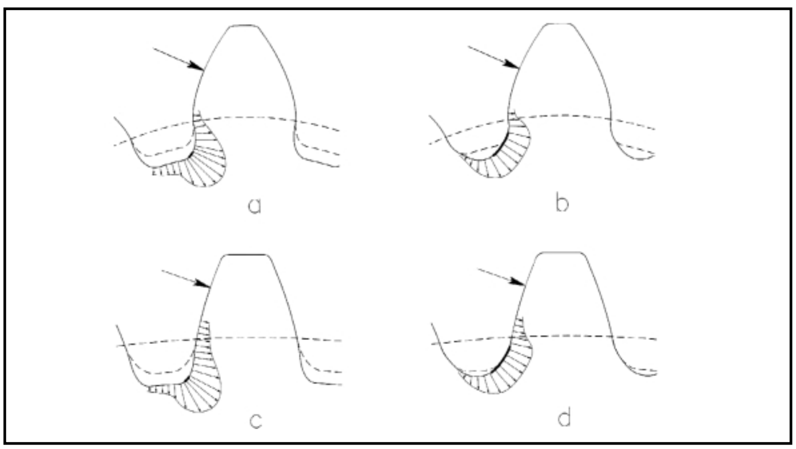

Fig. 4—Bending stress concentration for the balanced, traditionally designed (standard tool parameters) gear pair. Dashed line is the form circle; dotted line is the trajectory of the mating gear tooth; thick line is the stress concentration area; a and b are the 12-tooth pinion profiles before and after optimization, respectively; c and d are the 41-tooth gear profiles before and after optimization, respectively.

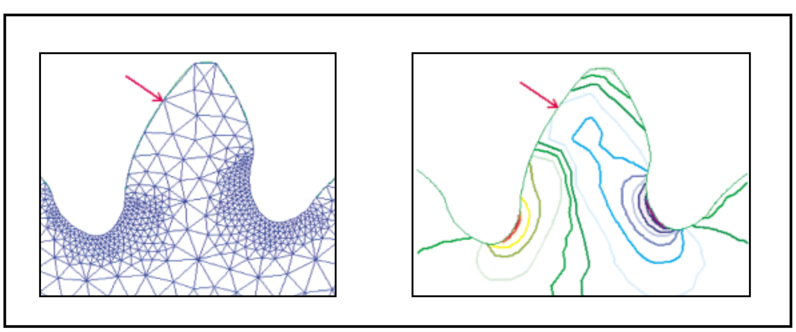

Fig. 5—FEA mesh and stress isograms for asymmetric tooth.

exceed 2%. The fillet shapes for these cases are also slightly different. Optimization of the pinion and the gear fillet profiles can result in bending stress differences exceeding the permissible balance tolerance δ_s. In this case, the tooth thickness ratio should be adjusted

$$C_{ttopt} = C_{tt} \bullet S_{max2}/S_{max1} \quad (3)$$

and the optimization process should be repeated.

Table 1 shows the results of the fillet optimization of gears designed by the traditional standard rack generative method. The generating rack profiles with 25° and 28° pressure angles provide a much lower level of maximum bending stress compared to the standard 20° generating rack. As a result, the fillet optimization of the high-pressure-angle gear tooth profiles gives less significant relative bending stress reductions than for standard 20° gear teeth. Figure 4 shows the bending stress concentration before and after fillet profile optimization. The optimized fillet has a more even bending stress distribution along its profile compared to the fillet of the standard rack-generated gear.

Fillet optimization provides maximum bending stress reduction for gears with asymmetric teeth (Refs. 5 and 6) as well. Optimized asymmetric-tooth FEA mesh and stress isograms are shown in Figure 5.

Tolerancing

The bending stress balance and the fillet optimization are calculated for the pair of gears at the tooth profiles' maximum material condition and absolute minimum (counting on maximum gears' runouts and their misalignments) center distance in the zero backlash mesh.

The specified center distance is:

$$\begin{aligned} a_{wmin} &= a_{wabs} + (ro_1 + ro_2)/2 + bl_{min}/\cos(\alpha_{wabs}) + \delta a_w \\ a_{wnom} &= a_{wabs} + \Delta a_w/2 \\ a_{wmax} &= a_{wnom} + \Delta a_w/2 \end{aligned} \quad (4)$$

where

a_{wabs} is the absolute minimum center distance that was used for bending stress balance and fillet optimization,

ro_1 and ro_2 are the operating pitch diameters' runouts of the pinion and the gear,

bl_{min} is minimum normal operating backlash,

α_{wabs} is the pressure angle that was used for bending stress balance and fillet optimization,

δa_w is the center distance variation related to other factors, like the maximum tooth alignment variation (including the shafts' misalignment), the bearing radial play, thermal expansion, etc.,

Δa_w is the total center distance tolerance.

The tooth parameters' tolerancing is shown in Figure 6.

The minimum material condition tooth thickness at the reference diameter is:

$$S_{w1,2\min} = S_{w1,2\max} - \Delta S_{wp1,2} \qquad (5)$$

where

$S_{w1,2\max}$ are the maximum material condition tooth thickness of the pinion or of the gear at the reference diameter,

$\Delta S_{wp1,2}$ is the tooth thickness tolerance.

The minimum material condition root and outside diameters are:

$$d_{r1,2\min} = d_{r1,2\max} - \Delta d_{r1,2} \qquad (6)$$

$$d_{a1,2\min} = d_{a1,2\max} - \Delta d_{a1,2} \qquad (7)$$

where

$d_{r1,2\max}$ and $d_{a1,2\max}$ are the maximum material condition root and outside diameters,

$\Delta d_{r1,2}$ and $\Delta d_{a1,2}$ are the tolerances for root and outside diameters.

Tool Profile Definition

The gear manufacturing process drives the tool design first of all. The most common gear manufacturing processes are gear machining and gear forming.

The gear machining process uses the copying or generating methods.

The copying gear machining method is used for milling, fly-cutting, shear-speed cutting, broaching, and form grinding. The tool profile is identical or very close to the space profile between neighboring teeth (for milling and fly-cutting) or the space around all gear teeth (for shear-speed cutting and broaching). It allows using the tooth space profile including the optimized fillet as the tool profile.

The generating gear machining method is used for hobbing, shaper-cutting, and generative grinding. The tool profile is not identical to the tooth space profile and can be defined by the reversed "gear forms tool" generating method (Fig. 7). The gear teeth with optimized fillet profile are set in mesh with a generated tool rack (for hobbing and generative grinding) or with a generated tool gear (for shaper-cutting). The tool pressure angle is selected to provide desirable machining conditions.

The gear forming process is typical for powder metal gears, plastic and metal injection molded gears, forged and extruded gears. The tool cavity profile (Fig. 8) looks similar to the gear profile, but it is adjusted for shrinkage. Shrinkage adjustment depends on the manufacturing method, process parameters, gear shape and material, etc.

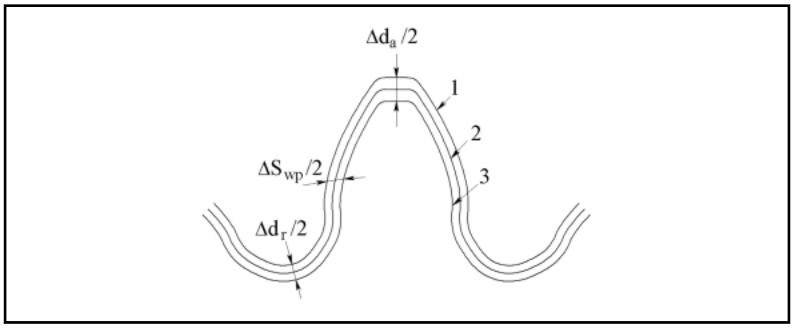

Fig. 6—Tooth parameters' tolerancing; 1, 2, and 3 are the tooth profiles at maximum, nominal, and minimum material conditions.

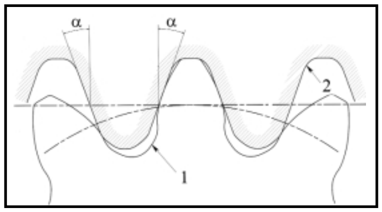

Fig. 7—Reversed "gear forms tool" rack-type tool profile generation; 1—the gear profile, 2— the tool profile.

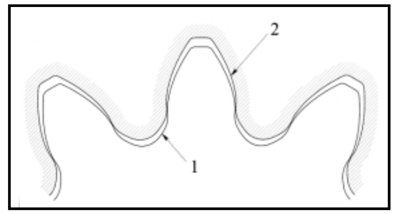

Fig. 8—Molding tool cavity profile definition; 1—the gear profile, 2—the cavity profile.

Summary

Direct Gear Design uses FEA for bending stress evaluation because the Lewis equation and its related coefficients do not provide a reliable solution to the wide variety of non-standard gear tooth profiles that could be considered.

Bending stress balance allows equalizing the tooth strength and durability for the pinion and the gear.

Optimization of the fillet profile allows reducing the maximum bending stress in the gear tooth root area by 10–30%. It works equally well for both symmetric and asymmetric gear tooth profiles. The bending stress reduction leads to:

- Size and weight reduction
- Longer life
- Higher load application
- Cost reduction (less expensive materials, heat treatment, etc.)
- Noise and vibration reduction, increased efficiency (finer pitch, more teeth will result in higher contact ratio for the given center distance).

The paper also describes an approach to the tooth parameters' tolerancing and tool profile definition.

References

1. Townsend, D.P. *Dudley's Gear Handbook*, McGraw-Hill, 1991.
2. Kapelevich, A.L., and R.E. Kleiss. "Direct Gear Design for Spur and Helical Gears," *Gear Technology*, September/October 2002, pp. 29–35.
3. *ANSI/AGMA 1006-A97—Tooth Proportions for Plastic Gears*, Appendix F: "Generating Gear Geometry without Racks," AGMA, Alexandria, VA, 1997.
4. Pulley, F.T., M.W. Kipling, G.A. Matson, D.L. Thurman and B.W. Avery. "Method for producing and controlling a fillet on a gear," US Patent #6164880, 2000.
5. Kapelevich, A.L. "Geometry and design of involute spur gears with asymmetric teeth," *Mechanism and Machine Theory*, 2000, Issue 35, pp. 117–130.
6. Litvin, F.L., Q. Lian and A.L. Kapelevich. "Asymmetric modified gear drives: reduction of noise, localization of contact, simulation of meshing and stress analysis," *Computer Methods in Applied Mechanics and Engineering*, 2000, Issue 188, pp. 363–390.

Tell Us What You Think . . .

Send e-mail to *people@geartechnology.com* to

- **Rate this article**
- **Request more information**
- **Contact the authors or organization mentioned**
- **Make a suggestion**

Or call (847) 437-6604 to talk to one of our editors!