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# AGMA Technical Paper

# Rating of Asymmetric Tooth Gears

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## Abstract

Nowadays there is a growing interest to application of asymmetric tooth gears in high performance unidirectional gear drives. About 30-40 years ago this type of gears was hardly known. Today situation is different, researchers and engineers from many countries are developing asymmetric tooth gear drives for different applications. There are numerous publications on this topic.
The benefits of gears with asymmetric tooth profiles are well known. The design objective of asymmetric tooth gears is to improve performance of the primary drive flank profiles at the expense of the opposite coast profiles' performance. The coast flanks are unloaded or lightly loaded during a relatively short work period. Asymmetric tooth profiles make it possible to simultaneously increase the contact ratio and operating pressure angle of drive tooth flanks beyond those limits achievable with conventional symmetric tooth gears. The main advantage of asymmetric tooth gears is drive flank contact stress reduction, which allows one to considerably amplify power transmission density, increase load capacity, and reduce size and weight. However, asymmetric tooth gears and their rating are not described by existing gear design standards.

This paper presents a rating approach for asymmetric tooth gears by their bending and contact stress levels in comparison with symmetric tooth gears, whose rating is defined by standards. This approach applies finite element analysis (FEA) for bending stress definition and the Hertz equation for contact stress definition. It defines equivalency factors for practical asymmetric tooth gear design and rating.

The paper illustrates the rating of asymmetric tooth gears with numerical examples.



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# RATING of ASYMMETRIC TOOTH GEARS

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## 1. Introduction

**Figure 1 – Asymmetric tooth gears**

This paper is motivated by the AKGears customers' request for asymmetric tooth gear design.
Although the gear geometry and design of asymmetric tooth gears (Figure 1) are known and described in a number of technical articles and books, they are not covered by modern national and international gear design and rating standards. This limits their broad implementation for various gear applications, despite substantial performance advantages in comparison to symmetric tooth gears for unidirectional drives. On the other side, asymmetric teeth, though nonstandard, have involute flanks like standard involute gears with symmetric teeth. Their drive and coast flank involutes unwind from two different base circles, and drive and coast pressure angles at a reference diameter are different. Typically (but not always), a drive tooth flank has a higher pressure angle than the coast flank. Although it leads to the drive flank contact ratio reduction, selection of the drive tooth flank with a higher pressure angle allows for reducing contact stress of the drive flanks and increasing gear transmission density of asymmetric tooth gears. An asymmetry factor that defines the difference between drive and coast pressure angles is a subject for optimization [2].
In some industries, like aerospace, which are accustomed to using gears with nonstandard tooth shapes, rating of these gears is established by comprehensive testing [1]. However, such testing programs are not affordable for many less demanding gear drives that could also benefit from asymmetric tooth gears. For such applications the asymmetric tooth gear design must be verified based of the existing gear rating standards. The rating approach presented in this paper is an attempt to resolve this issue and to bridge the gap between the stress evaluation methods of symmetric and asymmetric tooth gears and to allow for the application of existing rating standards to asymmetric tooth gears.

## 2. Design methods of asymmetric tooth gears

### 2.1. Traditional design of asymmetric tooth gears

Some researchers describe the geometry of asymmetric tooth gears by applying a traditional rack generating method [3]–[8]. This method defines asymmetric gear geometry by the preselected asymmetric generating gear rack parameters and addendum modifications (Figure 2). Typically, an asymmetric generating rack is modified from the standard symmetric rack by increasing the pressure angle of one flank. The opposite flank and other rack tooth proportions remain unchanged.

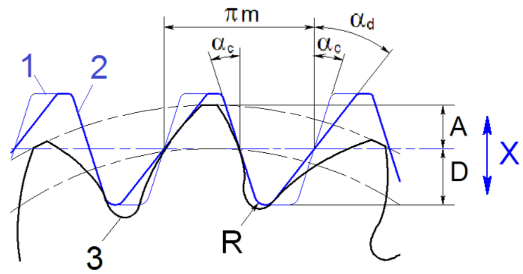



**Figure 2**

1 – initial standard symmetric generating rack;
2 – modified asymmetric generating rack; 3 – gear profile; A – gear addendum;
D – dedendum; X – addendum modification (X-shift); R – rack tip radius; m – module;
$\alpha_d$ – drive profile (pressure) rack angle; $\alpha_c$ – coast profile (pressure) rack angle

### 2.2. Direct design of asymmetric tooth gears

The alternative Direct Gear Design® method [9] does not limit gear parameter definition by a preselected generating rack, allowing comprehensive customization of asymmetric tooth geometry to maximize gear drive performance. This design method presents an asymmetric tooth by two involutes of two different base circles ($d_{bd}$ and $d_{bc}$) and a tooth tip circle $d_a$ (Figure 3).

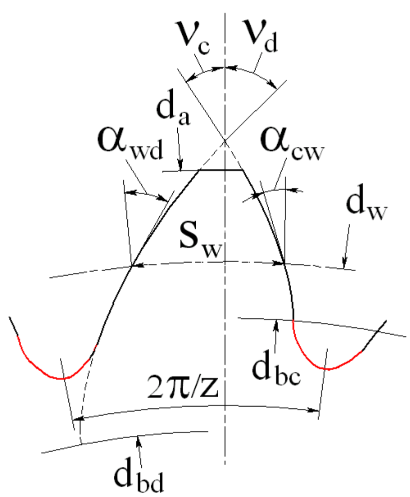



**Figure 3 – Tooth profile (root fillet profiles in red)**

$z$ – number of teeth; $d_{bd}$, $d_{bc}$ – base diameters; $\nu_d$, $\nu_c$ – involute intersection profile angles; $d_w$ – operating pitch diameter; $\alpha_{wd}$, $\alpha_{wc}$ – profile (pressure) angles at diameter; $d_w$, $S_w$ – circular tooth thickness at diameter; $d_w$, $d_a$ – tooth tip circle diameter; symbols "$d$" and "$c$" are for drive and coast tooth flanks

Drive and coast profile (pressure) angles $\alpha_d$ and $\alpha_c$ at operating pitch diameter $d_w$

$$\alpha_{wd} = \arccos\left(\frac{d_{bd}}{d_w}\right) \tag{1}$$

$$\alpha_{wc} = \arccos\left(\frac{d_{bc}}{d_w}\right) \tag{2}$$

Asymmetry factor $K$

$$K = \frac{d_{bc}}{d_{bd}} = \frac{\cos(\nu_c)}{\cos(\nu_d)} = \frac{\cos(\alpha_{wc})}{\cos(\alpha_{wd})} \geq 1.0 \tag{3}$$

Circular tooth thickness $S_w$ at operating pitch diameter $d_w$

$$S_w = \frac{d_w}{2}\left[\mathrm{inv}(\nu_d) + \mathrm{inv}(\nu_c) - \mathrm{inv}(\alpha_{wd}) - \mathrm{inv}(\alpha_{wc})\right] \tag{4}$$

Equally spaced teeth form the gear. The root fillet between teeth is the area of maximum bending stress. Direct Gear Design optimizes the root fillet profile, providing minimum bending stress concentration and sufficient clearance with the mating gear tooth tips in mesh [10], [11].

## 3. Comparable symmetric tooth gear definition

In order to apply existing rating standards to asymmetric tooth gear rating, the asymmetric tooth gears must be replaced by comparable symmetric tooth gears. Tooth geometry of these symmetric tooth gears should be described by symmetric generating rack parameters and addendum modifications (or X-shift coefficients).

### 3.1. Transformation of asymmetric generating rack to symmetric rack for comparable symmetric tooth gear generation

Traditional gear design of asymmetric tooth gears uses an asymmetric generating rack and addendum modifications. In order to define the tooth geometry of comparable symmetric tooth gears, the asymmetric generating rack should be transformed to the symmetric generating rack.

Parameters of this symmetric rack include (Figure 4):

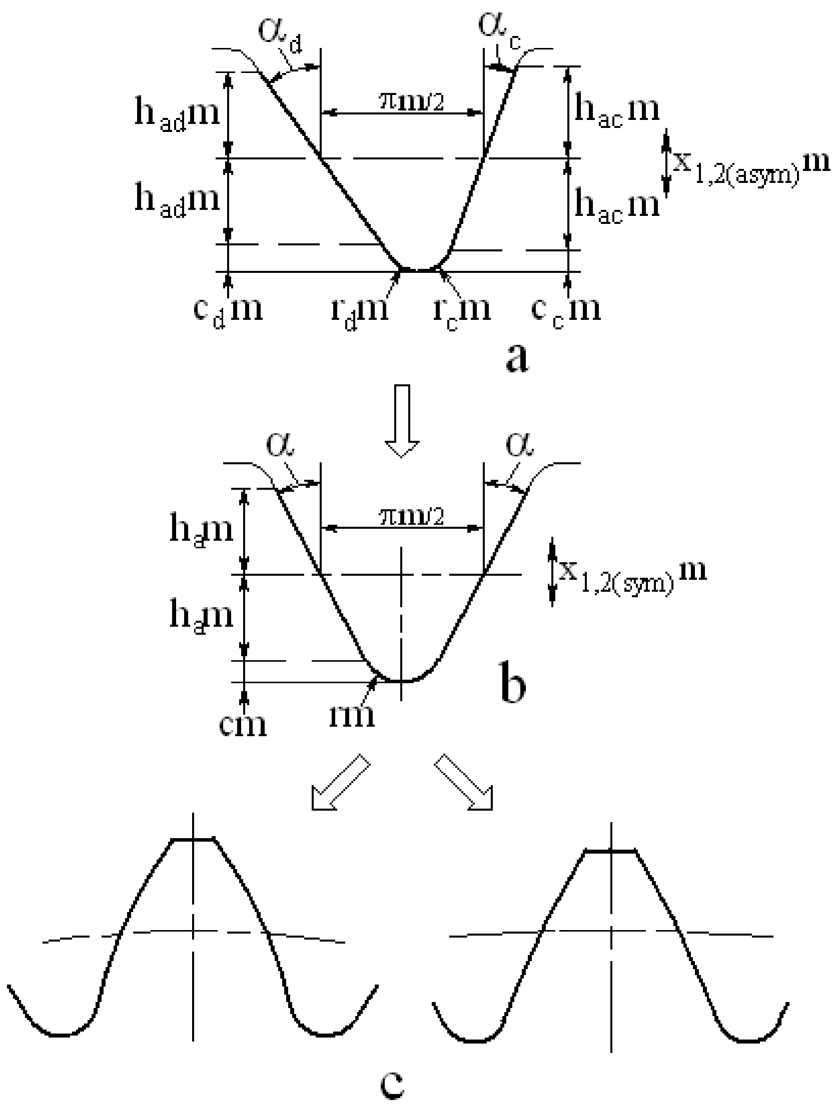



**Figure 4 – Transformation of asymmetric generating rack to symmetric rack for comparable symmetric tooth gear generation**
a – asymmetric rack; b – symmetric rack; c – comparable symmetric tooth profiles

Symmetric generating rack profile (pressure) angle

$$\alpha = \frac{\alpha_d + \alpha_c}{2} \tag{5}$$

Rack addendum coefficient

$$h_a = \frac{h_{ad} + h_{ac}}{2} \tag{6}$$

Full rack tip radius coefficient

$$r = \frac{\pi / 4 - h_a \tan\alpha}{\cos\alpha} \tag{7}$$

Clearance coefficient

$$c = r(1 - \sin\alpha) \tag{8}$$

Addendum modification (X-shift) coefficients

$$x_{1,2(\mathrm{sym})} = x_{1,2(\mathrm{asym})} \tag{9}$$

where index "1" and "2" are for the pinion and gear, respectively.

### 3.2. Definition of symmetric rack for comparable symmetric tooth gear generation based on Direct Gear Design of asymmetric tooth gear pair

Direct gear design of asymmetric tooth gears does not utilize any racks to generate gear tooth geometry parameters. However, in order to define the tooth geometry of comparable symmetric tooth gears that would be used for asymmetric tooth gear rating, the symmetric generating rack should be defined by asymmetric gear parameters.

Parameters of this symmetric rack include (Figure 5):

Symmetric generating rack module

$$m = \frac{d_{\mathrm{w1}}}{z_1} = \frac{d_{\mathrm{w2}}}{z_2} \tag{10}$$

where $z_1$ and $z_2$ are numbers of teeth of the pinion and gear, respectively.

Profile (pressure) angle

$$\alpha = \frac{\alpha_{\mathrm{wd}} + \alpha_{\mathrm{wc}}}{2} \tag{11}$$

Rack addendum coefficient

$$h_{\mathrm{a}} = \frac{d_{\mathrm{a1}} - d_1 + d_{\mathrm{a2}} - d_2}{4m} \tag{12}$$

Full rack tip radius coefficient

$$r = \frac{\pi / 4 - h_{\mathrm{a}} \tan\alpha_{\mathrm{w}}}{\cos\alpha_{\mathrm{w}}} \tag{13}$$

Clearance coefficient

$$c = r(1 - \sin\alpha_{\mathrm{w}}) \tag{14}$$

Addendum modification (X-shift) coefficients

$$x_1 = \frac{s_1 - s_2}{4m\tan\alpha} \text{ and } x_2 = -x_1 \tag{15}$$

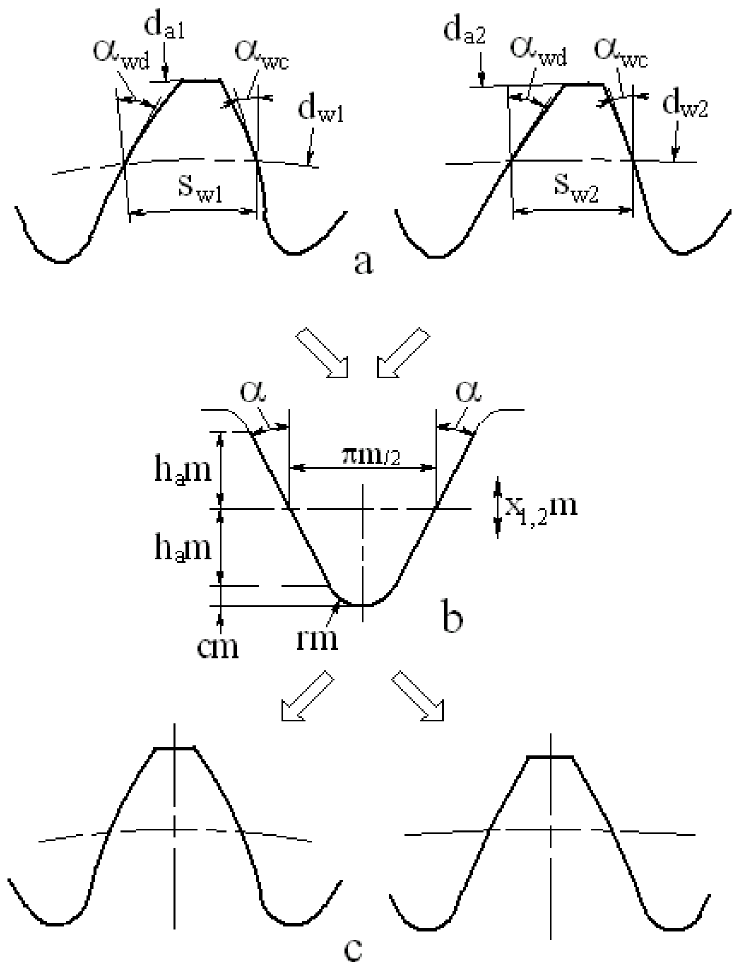


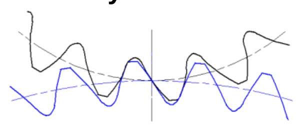
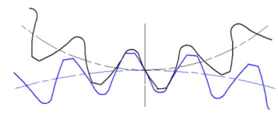

**Figure 5 – Definition of symmetric rack for comparable symmetric tooth gears generation based on Direct Gear Design of asymmetric tooth gear pair**

a – mating asymmetric tooth pinion and gear profiles;
b – symmetric rack; c – comparable symmetric tooth profiles

Depending on whether the asymmetric gear design method utilized is traditional or direct, the symmetric generating rack parameters defined by Equations 5–9 or 10–15 are used to design the comparable symmetric gears and obtain their rating data for required gear drive operating conditions.

A sample of the asymmetric and comparable symmetric tooth gear geometry data is presented in Table 1.

**Table 1 – Asymmetric and comparable symmetric tooth gear geometry data**

| **Gear Pair** | **Asymmetric**  | | **Comparable Symmetric**  | |
|---|---|---|---|---|
| Number of teeth | 20 | 49 | 20 | 49 |
| Module | 5.000 | | 5.000 | |
| Pressure Angle | 35º/20º* | | 27.5º | |
| Asymmetry Factor | 1.147 | | 1.0 | |
| Pitch Diameter (PD) | 100.000 | 295.000 | 100.000 | 295.000 |
| Base Diameter | 81.915/ 93.969* | 200.692/ 230.225* | 88.701 | 217.318 |
| Tooth Thickness at PD | 8.168 | 7.540 | 8.168 | 7.540 |
| Center Distance | 172.500 | | 172.500 | |
| Generating Rack Angle | - | | 27.5º | |
| Addendum Coefficient | - | | 0.951 | |
| Root Radius Coefficient | - | | 0.327 | |
| Root Clearance Coefficient | - | | 0.176 | |
| Profile Shift Coefficient | - | - | 0.060 | -0.060 |
| Tip Diameter | 109.802 | 254.214 | 110.110 | 253.910 |
| Root Diameter | 89.080** | 233.597** | 89.360 | 233.141 |
| Root Fillet Profile | optimized | optimized | trochoidal | trochoidal |
| Face Width | 30.00 | 27.00 | 30.00 | 27.00 |
| Contact ratio | 1.20/1.55* | | 1.31 | |

*drive/coast flanks

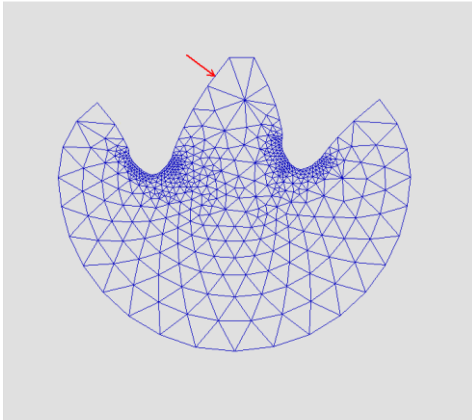
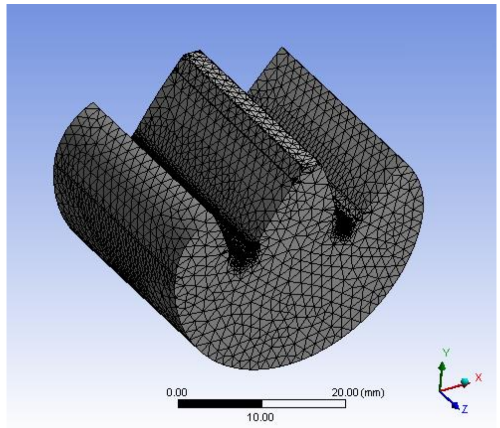
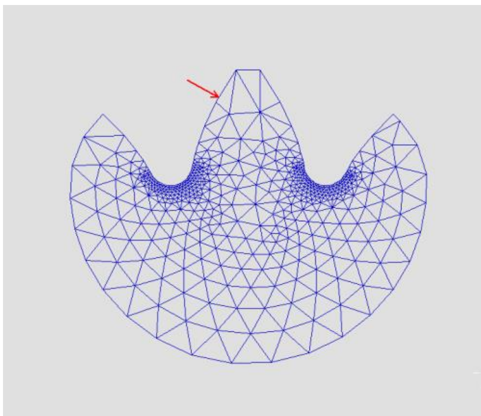
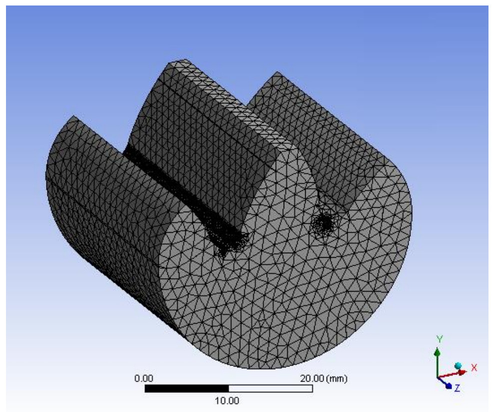
**root fillet optimized

## 4. Stress calculation of asymmetric and comparable symmetric tooth gears

### 4.1. Root bending stress and conversion coefficients

The standard procedure for bending stress calculation (based on the Lewis equation) cannot be used for asymmetric tooth gears because a symmetric Lewis parabola does not properly fit into an asymmetric tooth profile. Finite Element Analysis (FEA) is a more suitable analytical tool to calculate the maximum root stress in the asymmetric and comparable symmetric tooth gears in order to define bending stress conversion coefficients. The Direct Gear Design technique utilizes the FEA tooth root bending stress calculation for both symmetric and asymmetric tooth gears [9]. Correlations between standard and FEA root stress were explored by Vanyo Kirov [12]. Although there are differences in the standard and FEA root stress calculation results, FEA allows for defining conversion coefficients between asymmetric and comparable symmetric tooth maximum bending stresses. A 2D or 3D FEA program can be used for tooth root bending stress calculations. This article describes the 2D FEA procedure developed by Yuriy Shekhtman. ANSYS software was used for the 3D FEA. The 2D and 3D finite element meshes of the asymmetric and comparable symmetric gear teeth are shown in Table 2.

**Table 2 – 2D and 3D finite element meshes of asymmetric and comparable symmetric teeth**

| | 2D mesh | 3D mesh |
|---|---|---|
| **Asymmetric tooth** |  |  |
| **Comparable symmetric tooth** |  |  |

For the maximum root bending stress calculation, a normal load $F_n$ is applied to the Highest Point of Single Tooth Contact (HPSTC) of the drive tooth flank.

$$F_n = \frac{2T_1}{d_{b1}} \tag{16}$$

where $T_1$ is the pinion driving torque, $d_{b1}$ is the pinion base diameter.

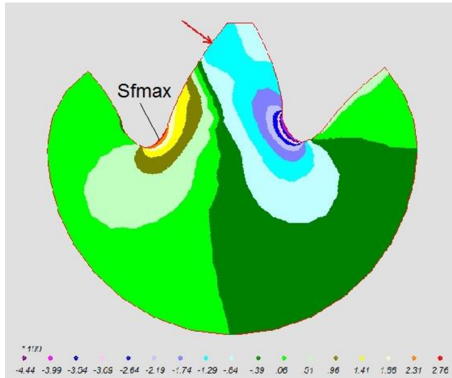
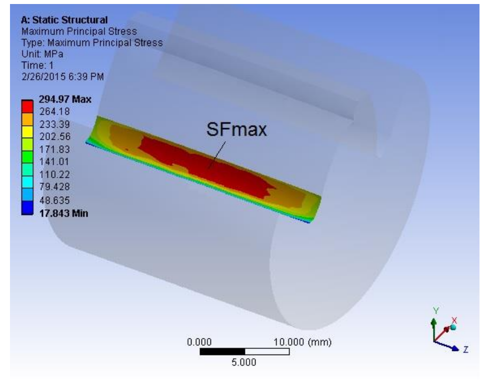
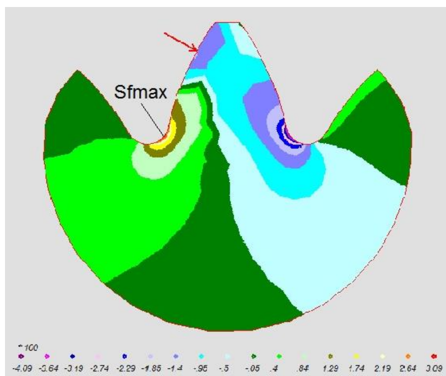
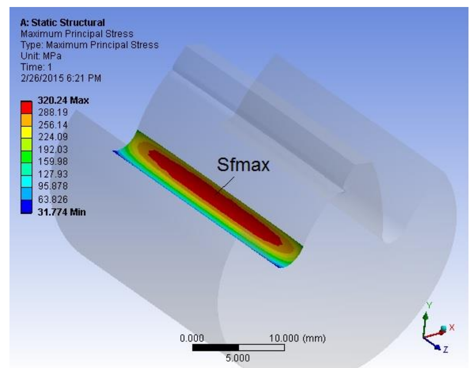
The pinion and gear conversion coefficients are

$$C_{F1,2} = \frac{\sigma_{Fmax(sym)1,2}}{\sigma_{Fmax(asym)1,2}} \tag{17}$$

where $\sigma_{Fmax(asym)1,2}$ and $\sigma_{Fmax(sym)1,2}$ are the maximum FEA root bending stresses of the asymmetric and comparable symmetric tooth pinion and gear.

2D and 3D finite element stress models of the asymmetric and comparable symmetric gear teeth are shown in Table 3.

**Table 3 – Root fillet stress of asymmetric and comparable symmetric teeth**

| | 2D model | 3D model |
|---|---|---|
| **Asymmetric tooth** |  |  |
| **Comparable symmetric tooth** |  |  |

The standard tooth flank contact stress calculation procedure (based on the Hertz equation) is suitable for both symmetric and asymmetric tooth gears.

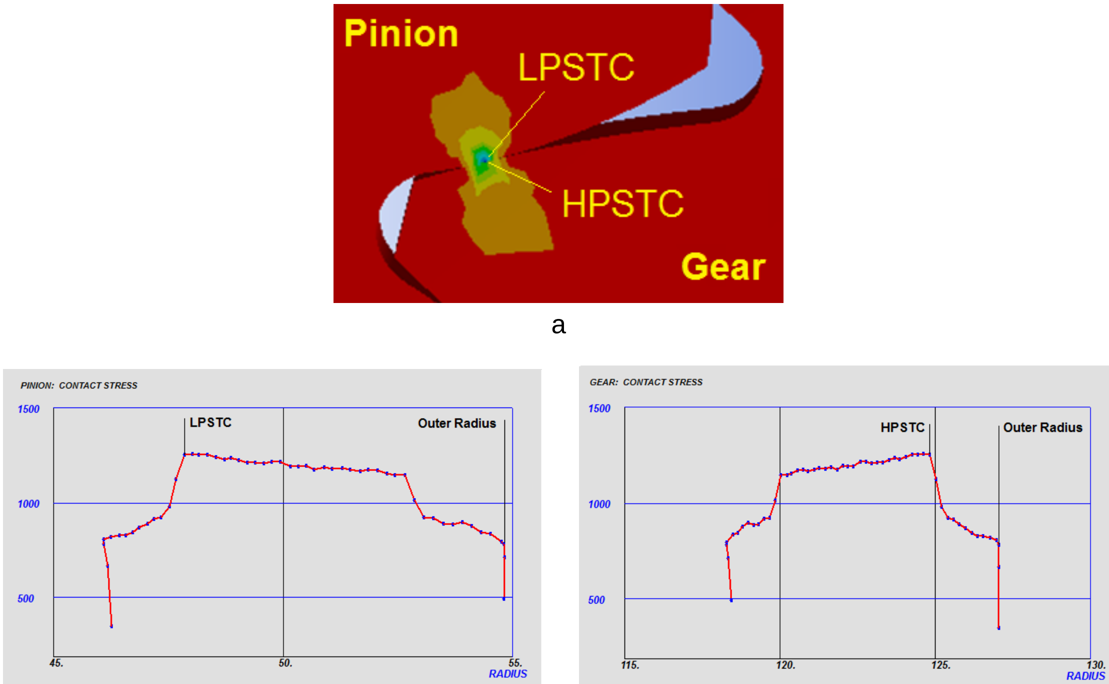
The Hertz equation allows for calculating the maximum contact stress in asymmetric and comparable symmetric tooth gears to define the contact stress conversion coefficients.

The Hertzian contact stress is

$$\sigma_H = \sqrt{\left(\frac{F_n}{\pi b}\right)\left(\frac{E}{2(1-\nu^2)}\right)\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)} \qquad (18)$$

where $b$ is face width in contact, $E$ and $\nu$ are modulus of elasticity and Poisson ratio, assuming mating pinion and gear materials are identical, $\rho_1$ and $\rho_2$ are pinion and gear curvature radii in contact.

For a spur pinion and gear with a contact ratio < 2.0, the maximum flank contact stress is localized near the Lowest Point of Single Tooth Contact (LPSTC) of the drive tooth flank of the pinion. The pinion drive flank LPSTC point coincides with the gear drive flank HPSTC point (Figure 6).




a

b c

**Figure 6 – Contact stress point**

a – mating pinion and gear contact point; b and c – contact stress charts of pinion and gear

The contact stress conversion coefficient is

$$C_{\mathrm{H}} = \frac{\sigma_{\mathrm{Hmax(sym)}}}{\sigma_{\mathrm{Hmax(asym)}}} \tag{19}$$

where $\sigma_{\mathrm{Hmax(asym)}}$ and $\sigma_{\mathrm{Hmax(sym)}}$ are the maximum Hertz contact stresses of the asymmetric and comparable symmetric tooth gears pairs.

## 5. Standard rating of asymmetric tooth gears

Rating of involute gears with symmetric tooth gears is established in national and international standards [13], [14]. In order to apply these rating standards to asymmetric tooth gears, the bending and contact safety factors defined for the comparable symmetric tooth gears should be multiplied by the contact and bending conversion coefficients accordingly. Then the rated bending safety factors of asymmetric tooth gears are

$$S_{\mathrm{F(asym)1,2}} = C_{\mathrm{F1,2}} S_{\mathrm{F(sym)1,2}} \tag{20}$$

where $S_{\mathrm{F(sym)1,2}}$ are the root bending safety factor of comparable symmetric tooth gears defined by the rating standards.

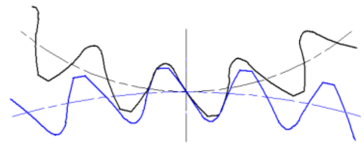
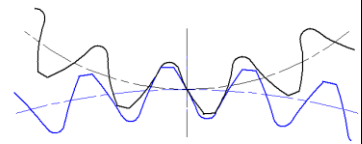
The rated contact safety factor of asymmetric tooth gears is

$$S_{\mathrm{H(asym)}} = C_{\mathrm{H}} S_{\mathrm{H(sym)}} \tag{21}$$

where $S_{\mathrm{H(sym)}}$ is the flank contact safety factor of comparable symmetric tooth gears defined by the rating standards.

A sample of the asymmetric and comparable symmetric tooth gear stress analysis results is presented in Table 4. Geometric data for these gears is in Table 3.

**Table 4 – Asymmetric and comparable symmetric tooth gear stress analysis results**

| **Gear Pair** | **Asymmetric**  | | **Comparable Symmetric**  | |
|---|---|---|---|---|
| Number of teeth | 20 | 49 | 20 | 49 |
| Module | 5.000 | | 5.000 | |
| Pressure Angle | 35º/20º* | | 27.5º | |
| Torque, Nm | 900 | 2205 | 900 | 2205 |
| RPM | 1000 | 408 | 1000 | 408 |
| Service Life, hours | 2000 | | 2000 | |
| Material type | Carburized, case harden steel, like AISI 8620 | | | |
| Bending Stress (2D FEA), MPa | 276 | 277 | 309 | 334 |
| Bending Stress (3D FEA), MPa | 295(+7%) | 284(+2.5%) | 320(+3.5%) | 350(+5%) |
| Bending Stress, MPa | - | - | 448* | 480* |
| Contact Stress, MPa | - | - | 1507* | 1407* |
| Maximum Contact Stress, MPa | 1257 | | 1349 | |
| Bending Stress Conversion Coefficients (2D FEA), $C_{F1,2}$ | 1.120 | 1.206 | - | - |
| Bending Stress Conversion Coefficients (3D FEA), $C_{F1,2}$ | 1.085 | 1.232 | - | - |
| Contact Stress Conversion Coefficients (Hertz), $C_H$ | 1.073 | | - | |
| Bending Safety Factors | 1.90/1.84** | 1.95/2.00** | 1.70* | 1.62* |
| Contact Safety Factors | 1.02 | 1.12 | 0.95* | 1.04* |

*Calculation method: per ISO 6336 standard

**2D/3D FEA

## Summary

**1.** The article outlines a simple and effective approach to rating asymmetric tooth gears using existing symmetric tooth gear rating standards that includes:

- conversion of the asymmetric tooth geometry into the comparable symmetric tooth geometry and definition of its generating rack;
- calculation of maximum bending stresses using the 2D or 3D FEA to both asymmetric and comparable symmetric gear teeth;
- calculation of maximum contact stresses for both asymmetric and comparable symmetric gear teeth using the Hertz equation;
- definition of the bending and contact stress conversion coefficients;
- standard stress analysis for the comparable symmetric gear tooth and definition of the contact and bending safety factors;
- definition of the contact and bending safety factors for asymmetric tooth gears using the symmetric tooth gear safety factors and the bending and contact stress conversion coefficients.

**2.** The presented asymmetric tooth gear rating approach allows expanding implementation of these types of gears in many unidirectional gear drives, maximizing their performance.

**3.** This approach might be a temporary solution until the asymmetric tooth gear design standards will be developed by AGMA or ISO. However, it takes long time to create a new gear rating standard. Meanwhile, the suggested approach can be used today for rating of the asymmetric tooth gears.

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