

Direct Gear Design[®] for Automotive Applications

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ABSTRACT

This paper presents Direct Gear Design – an alternative method of analysis and design of involute gears which separates gear geometry definition from tool selection to achieve the best possible performance for a particular product and application. This method has successfully been applied for a number of automotive applications. Some examples will be presented at the end of the paper.

INTRODUCTION

The direct design approach, which uses the operating conditions and performance parameters as a foundation for the design process, is common for most parts of mechanisms and machines (for example, crankshafts, rods, etc.).



Fig. 1 These crankshaft and rods are examples of direct design

Ancient engineers successfully used Direct Gear Design. They knew the desired gear ratio and center distance, and available power source (e. g. water current, wind, horse power). They used them to define the gear parameters: diameters, number and shape of the teeth for each gear. Then they manufactured gears and carved their teeth using available materials, technology, and tools.

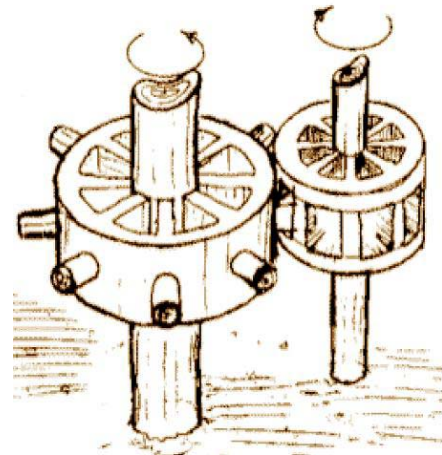


Fig.2 Ancient Direct Gear Design

It is important to note that the gear and tooth geometry were defined (or designed) first. The manufacturing process and tools then formed or cut this geometry in wood, stone, or metal. In other words, gear parameters were primary and the manufacturing process and tool parameters were secondary. This is the essence of Direct Gear Design.

During the technological revolution of the 19th century, the gear generating process was developed. This process uses a gear rack profile as a cutting edge of the hob that is in mesh with the gear blank.

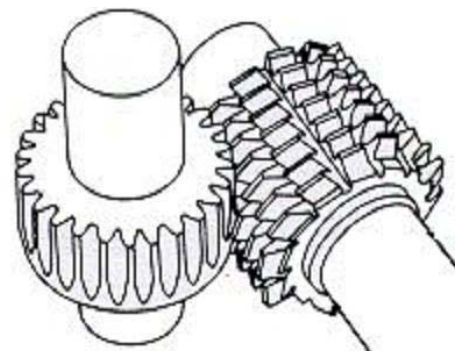


Fig.3 Gear hobbing

Gear hobbing is a reasonably accurate and highly productive manufacturing process. With some exceptions, gears that are cut by the same tool can mesh together. Hobbing machines require complicated and expensive tools. Common parameters of the cutting tool (generating rack) such as the profile (pressure) angle, diametral pitch, tooth addendum and dedendum (Fig.4) were standardized and have become the foundation for gear design. This has made gear design **indirect**, depending on the pre-selected (typically standard) set of cutting tool parameters.

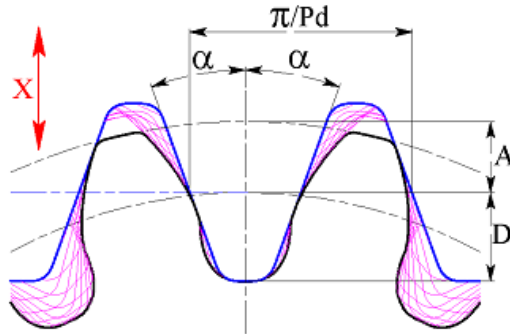


Fig.4 Generating rack parameters

This “traditional” gear design approach has its benefits including:

- Interchangeability of the gears
- Low tool inventory
- Simple gear design process

Once the tool is chosen, the only way to affect the gear tooth profile is to change the position of the tool relative to the gear blank. This will change the tooth thickness, root diameter, outer diameter, and strength of the tooth. This tool positioning is called addendum modification or X-shift. It is used to balance the gear strength and reduce sliding.

Table 1 presents a typical drawing specification, describing a traditionally designed spur gear. Most of its parameters belong to the tool or generating process parameters. Very few parameters actually belong to the gear.

Table 1

	Tool or Generating Process Parameter	Gear Parameter
NUMBER OF TEETH		X
STANDARD NORMAL PITCH	X	
PRESSURE ANGLE	X	
STANDARD PITCH DIAMETER	X	

BASE DIAMETER		X
ADDENDUM MODIFICATION (X-SHIFT)	X	
FORM DIAMETER	X	
ROOT DIAMETER		X
OUTSIDE DIAMETER		X
TOOTH THICKNESS ON STANDARD PITCH DIAMETER	X	
ADDENDUM	X	
WHOLE DEPTH	X	

Traditional gear design based on standard tool parameters provides “universality” which is acceptable for many gear applications. However, it does not provide the best possible performance for any particular gear application because it is constrained by predefined tooling parameters.

Traditional tool based gear design is not the only available approach to design gears. There is another approach - Direct Gear Design.

The theoretical foundation of modern Direct Gear Design was developed by Dr. E.B. Vulgakov in his Theory of Generalized Parameters [1]. Practical engineering implementation of this theory was called Direct Gear Design [2]. **Direct Gear Design is an application driven gear development process with primary emphasis on performance maximization and cost efficiency without concern for any predefined tooling parameters.**

The Direct Gear Design method typically includes:

- Gear Mesh Synthesis
- FEA Modeling, Load Sharing, and Stress Calculation
- Efficiency Maximization
- Bending Stress Balance
- Fillet Profile Optimization

1. GEAR MESH SYNTHESIS

1.1. Gear Tooth

Direct Gear Design defines the gear tooth without using the generating rack parameters like diametral pitch, module, or pressure angle. The gear tooth (Fig.5) is defined by two involutes of the base circle d_b and the circular distance (base tooth thickness) S_b between them. The outer diameter d_a limits tooth height to avoid having a pointed tooth tip and provides a desired tooth tip thickness S_a . The non-involute portion of the tooth profile, the fillet, does not transmit torque, but is a critical element of the tooth profile. The fillet is the area with the maximum bending stress, which limits the strength and durability of the gear.

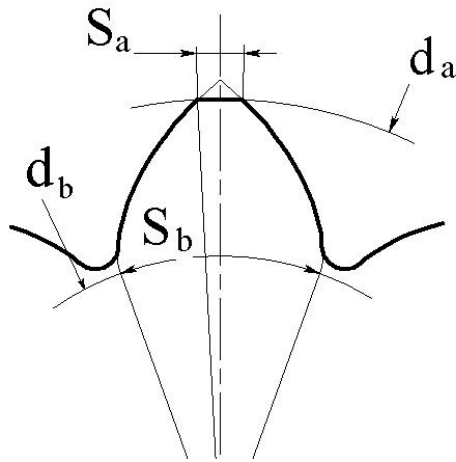


Fig.5 Tooth parameters

1.2. Gear Mesh

Two involute gears can mesh together (Fig.6) if they have the same base circle pitch.

Other parameters of a gear mesh are:

- Center distance a_w
- Operating pitch diameters d_{w1} and d_{w2} (diameters with pure rolling action and zero sliding)
- Tooth thicknesses on the operating pitch diameters S_{w1} and S_{w2}
- Operating pressure angle α_w (involute profile angle on the operating pitch diameters)
- Contact ratio ϵ_α

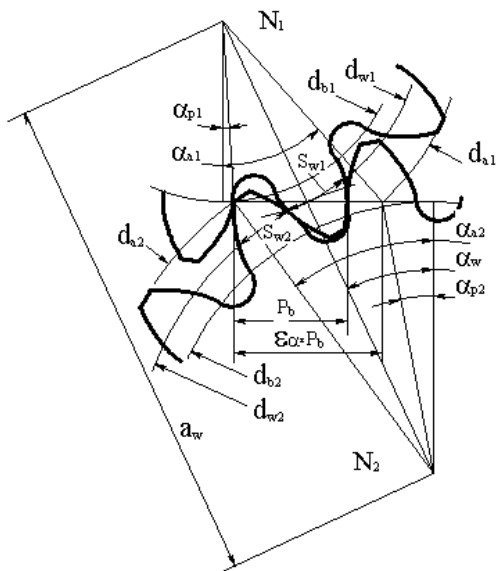


Fig.6 Mesh parameters

There is a principal difference in the pressure angle definitions in traditional and Direct Gear Design. In traditional gear design, the pressure angle is the tooling rack profile angle. In Direct Gear Design, the pressure angle is the mesh parameter. It does not belong to one gear. If the mesh condition (the center distance, for example) changes, the pressure angle changes as well.

1.3. Conversion to Spur Gear Mesh

Direct Gear design is applicable for all kinds of involute gears: spur, helical, bevel, worm, etc.

In order to optimize the tooth geometry of non-spur gears, the real pair of gears is converted to the virtual spur pair of gears. The virtual spur gears have the identical tooth profiles as the real gears in normal section, but with a different number of teeth. There is an assumption that the relative performance improvement of the virtual spur gears leads to an improvement of the real gears.

The following formulas are used to define the virtual numbers of teeth:

- Helical gears: $N_v = N/\cos^3(\beta)$, N is the real number of teeth, β is the operating helix angle
- Straight bevel gears: $N_v = N/\cos(\gamma)$, γ is the operating cone angle
- Spiral bevel gears: $N_v = N/\cos(\gamma)/\cos^3(\beta)$
- Worm gears: $N_{wv} = N_w/\cos^3(90^\circ - \beta)$ and $N_{wgv} = N_{wg}/\cos^3(\beta)$; N_w is the number of starts of the worm, N_{wg} is the number of teeth of the worm gear

1.4. Direct Gear Design Input Data

Unlike in traditional gear design (where input data includes the pre-selected, typically standard cutting tool parameters), in Direct Gear Design all input data are related to the gears or the gear mesh at nominal operating conditions.

Data, describing physical size of the gears and their teeth are:

Number of Teeth for the Pinion and the Gear
Nominal Operating Diametral Pitch or Nominal Operating Module (for metric gears)

Tooth configuration parameters are:

Nominal Operating Pressure Angle (for gears with asymmetric teeth the nominal operating pressure angles are different for drive side and coast side of the teeth).

Top Land Coefficients for the Gear and the Pinion - The top land coefficient is the nominal top land multiplied by the Nominal Operating Diametral Pitch or divided by Nominal Operating Module (for metric gears)

Nominal Contact Ratio (for drive side of the teeth) – can be used in input data instead of the Top Land Coefficients.

Preliminary Tooth Thickness Ratio - is the ratio between the pinion and the gear tooth thicknesses at the nominal pitch diameters. This parameter affects the top land size distribution between the pinion and the gear. The final Tooth Thickness Ratio can also be defined by the bending stress balance procedure.

Tip Radius Coefficients for the Gear and the Pinion - The tip radius coefficient is the nominal tip radius multiplied by the Nominal Operating Diametral Pitch or divided by the Nominal Operating Module (for metric gears). The tip radius coefficients are typically equal, but

could be different for the pinion and the gear and for the drive and coast sides of the tooth.

Material properties, friction condition and load parameters include:

Modulus of Elasticity and Poisson Ratio
Friction Coefficient
Pinion Torque

The dimensional tolerances include:

Minimum Backlash
Center Distance Tolerance
Tooth Thickness Tolerance for the Gear and the Pinion
Pitch Diameter Runout for the Gear and the Pinion

The FEA setting parameters:

Number of Tooth Profile Nodes - This number does not include the constrained nodes and the nodes inside the area limited by the tooth profile and constrained nodes;

Compression coefficient - is the ratio of the density of the fillet nodes to the density of other profile nodes (excluding the fillet nodes);

Fillet Optimization Parameters – the parameters that define the iteration and random search processes.

1.5. Direct Gear Design Output Data

The output data presents all nominal gear geometry parameters (diameters, profile angles, and tooth thicknesses, etc.), specific sliding velocities, gear efficiency, and geometrical and load data for the FEA. The tolerance output file includes the minimum and maximum values of the gear drawing specification parameters. The fillet profile and root diameter are initially defined and will be finalized after the fillet profile optimization operation.

2. FEA MODELING, LOAD SHARING, AND STRESS CALCULATION

The Direct Gear Design approach results in a wide variety of tooth profiles, depending on the particular gear drive performance priorities. For this reason, the Lewis equation and experimentally defined stress concentration factors, traditionally used for bending stress calculation of rack-generated gears, do not provide reliable results for direct designed gears. FEA is chosen as the Direct Gear Design stress analysis tool for bending stress and deflection calculations. For calculating contact stress and deflection, the Hertz equation is used. The load sharing operation defines the force distribution between the simultaneously meshed pairs of teeth, and calculates bending and contact stress in every phase of the gear mesh. FEA and the Hertz equation are used here in combination to define bending and contact deflection. Fig. 8 presents the typical load sharing, bending and contact stress charts for conventional and high contact ratio gears.

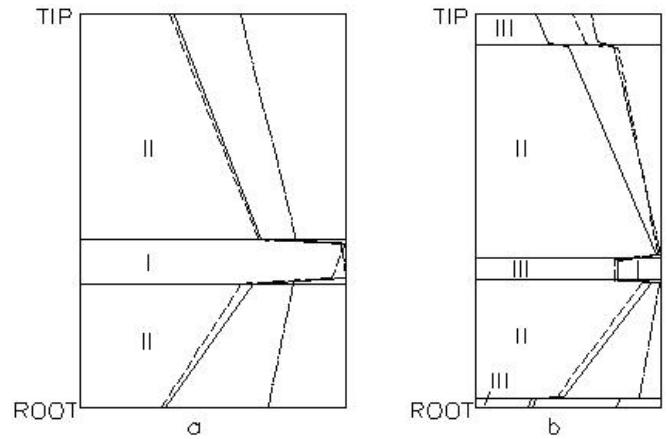


Fig. 7. Load sharing. a. conventional gears; b. high contact ratio gears, solid line – load distribution along the tooth; dash line – contact stress along the tooth; dashdot line – maximum bending stress at the tooth fillet, I – one pair of tooth mesh area; II – two pairs of tooth mesh area; III – three pairs of tooth mesh area.

Fig. 8 presents an FEA tooth model and a bending stress isograms chart.

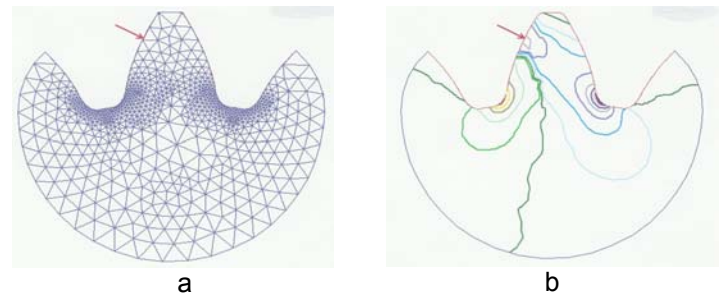


Fig. 8. a. The FEA mesh; b. the bending stress isograms.

3. EFFICIENCY MAXIMIZATION

In gear transmissions, almost all inefficiency or mechanical losses are transferred into heat, reducing gear performance, reliability, and life. This is especially critical for plastic gears. Plastics do not conduct heat as well as metal. Heat accumulates on the gear tooth surface, leading to premature failure.

The well-known [3] gear efficiency equation for spur gears is:

$$E := 100 \cdot \left[1 - \frac{f}{2 \cdot \cos(\alpha)} \cdot \frac{(H_1)^2 + (H_2)^2}{H_1 + H_2} \right] \%$$

Where:

H_1 and H_2 are the maximum specific sliding velocities of the pinion and the gear;

f is the friction coefficient;

α is the operating pressure angle.

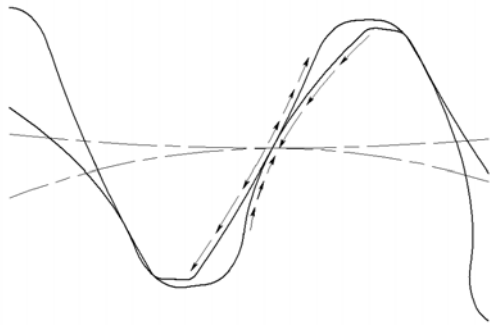


Fig.9 Sliding velocities directions

Direct Gear Design maximizes gear efficiency by equalizing the maximum specific sliding velocities for both gears (Fig. 9). Unlike in traditional gear design, it can be done without compromising gear strength or stress balance.

4. BENDING STRESS BALANCE

Mating gears should be equally strong. If the initially calculated bending stresses for the pinion and the gear are significantly different, the bending stresses should be balanced [4].

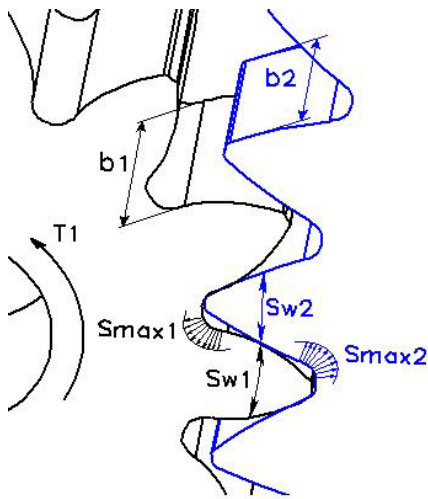


Fig.10 Balance of the maximum bending stresses

Direct Gear Design defines the optimum tooth thickness ratio S_{p1}/S_{p2} (Fig.10), using FEA and an iterative method, providing a bending stress difference of less than 1%. If the gears are made out of different materials, the bending safety factors should be balanced.

5. FILLET PROFILE OPTIMIZATION

Traditional gear design is based on predefined cutting tool parameters; the fillet is defined by the trace of the cutting tool's edge. The cutting tool typically provides a fillet profile with an increased radial clearance in order to avoid root interference, resulting in high teeth with large radial clearance and small fillet radii in the area of maximum bending stress.

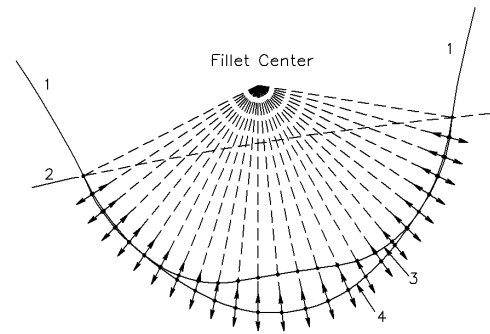
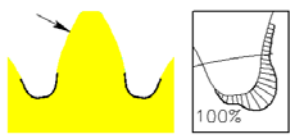
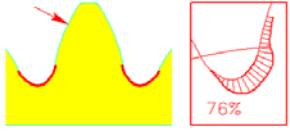


Fig.11 Fillet profile optimization; 1. involute profiles; 2. form diameter; 3. initial fillet profile; 4. optimized fillet profile.

Direct Gear Design optimizes the fillet profile for any pair of gears in order to minimize the bending stress concentration [4]. The initial fillet profile is a trace of the mating gear tooth tip. The optimization process is based on FEA and a random search method (Fig.11). The Direct Gear Design software program sets up the center of the fillet and connects it with the FEA nodes on the fillet. Then it moves all the nodes along the beams and calculates the bending stress. The nodes cannot be moved above the initial fillet profile because it will lead to interference with the mating gear tooth. The program analyzes successful and unsuccessful steps, altering the fillet profile to reduce the maximum bending stress. This process continues for a certain number of iterations, resulting in the optimized fillet profile.

Table 2 illustrates fillet profile optimization and the achievable maximum bending stress reduction for standard (AGMA 201.2) gears.

	Pinion	Gear
Diametral Pitch	10	
Pressure Angle	25°	
Number of teeth	10	10
Face Width	.500	.500
Torque, in-lb	200	
Results		
Fillet Profile	Tooth Profile	Bending Stress
Standard		12,800 psi
Optimized		9,800 psi

An example of gears with an optimized fillet profile is shown in Fig.12

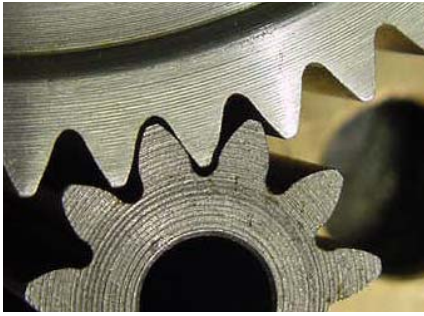


Fig.12 Gears with optimized fillets

6. GEARS WITH ASYMMETRIC TEETH

The two profiles (sides) of a gear tooth are functionally different for many gear drives. The workload on one profile is significantly higher and is applied for longer periods of time than for the opposite one. The design of an asymmetric tooth shape reflects this functional difference (Fig.13).

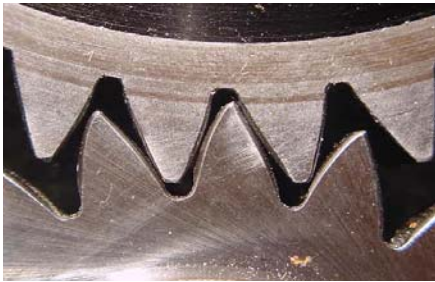


Fig.13 Gears with asymmetric teeth

The design intent of asymmetric gear teeth is to improve the performance of the primary contacting profile by degrading the performance of the opposite profile [5-8]. The opposite profile is typically unloaded or lightly loaded during relatively short work periods. The degree of asymmetry and drive profile selection for these gears depends on the application.

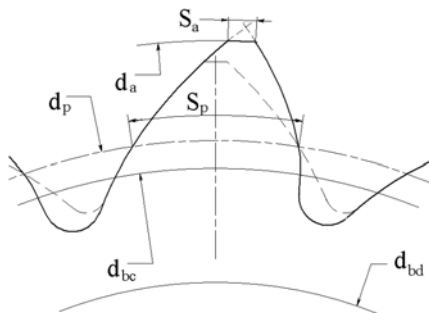


Fig.14 Asymmetric tooth parameters

The Direct Gear Design approach for asymmetric gears is the same as for symmetric gears. The only difference is that the asymmetric tooth (Fig.14) is defined by two involutes of two different base circles d_{bd} and d_{bc} . The common base tooth thickness does not exist in the

asymmetric tooth. The circular distance (tooth thickness) S_p between involute profiles is defined at some reference circle diameter d_p that should be bigger than the largest base diameter.

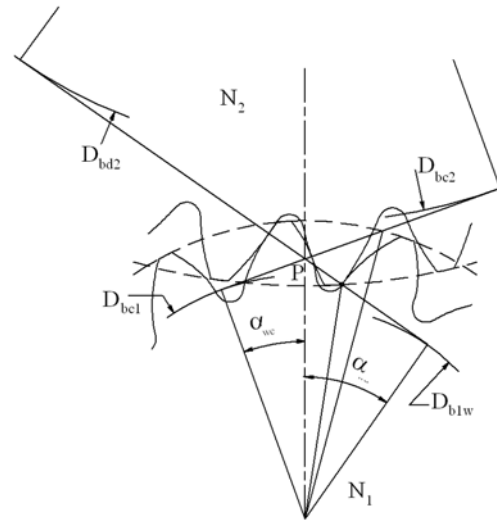


Fig.15 Asymmetric gear mesh

With asymmetric gears it is possible to simultaneously increase the transverse contact ratio and operating pressure angle beyond the conventional gear limits. For example, if the theoretical maximum pressure angle for symmetric spur involute gears is 45° pressure, the asymmetric spur gears can operate with pressure angles 50° or higher. Asymmetric gear profiles also make it possible to manage tooth stiffness and load sharing while maintaining the desired pressure angle and contact ratio on the drive profiles by changing the coast side profiles. These results in higher load capacity and lower noise and vibration levels compared with conventional symmetric gears.

7. TOOLING AND PROCESSING OF DIRECT DESIGNED GEARS

The Direct Gear Design approach is dedicated to custom gears and requires custom tooling. For cut metal gears, this means that every gear needs its own hob or shaper. This leads to increased gear cutting tool inventory and higher cost. The application of Direct Gear Design must be justified by a significant improvement in gear performance.

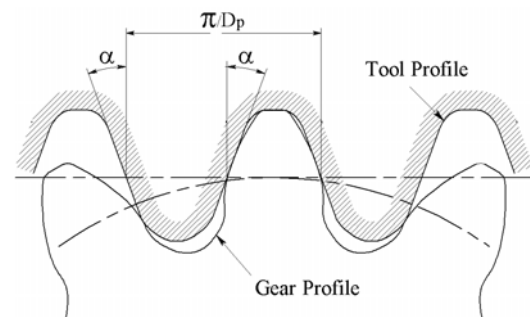


Fig.16 Cutting tool profile

The gear profile is in mesh with the tool, forming its cutting edge. This can be done at different mesh conditions, such as different pitch diameters and pressure angles. Typically the generating process is used to define the rack parameters with the closest standard diametral pitch or module. Then the tool pressure angle and other profile parameters are calculated. It allows for the use of standard hobs and for the regrinding of the cutting edge profile instead of making a whole new tool. The selected tool profile must satisfy the cutting condition requirements. The gear machining process for Direct Designed gears (including gears with asymmetric teeth) is practically the same as that for standard gears.

The plastic gear molding process (as well as gear die casting, gear forging, powder metal gear processing, etc.) does not use mesh generation and requires unique tooling for every gear. This makes Direct Gear Design naturally suitable for plastic molded gears, as customization of the gear tooth profile does not affect the tooling cost, delivery time, or gear processing time.

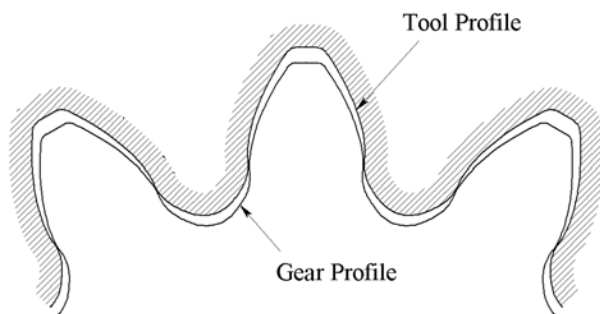


Fig. 17 Molded gear tool profile

The profile of a plastic gear tool cavity (Fig. 17) depends on many factors such as the gear shape, material properties, material shrinkage, the size and location of the gates, and the molding process parameters.

8. TRADITIONAL VS. DIRECT GEAR DESIGN

Table 3 illustrates the differences in basic principles and application of Traditional and Direct Gear Design.

Table 3

Traditional Gear Design	Direct Gear Design
<p>The diagram shows a gear tooth profile with a pressure angle α and a pitch diameter Pd. The tooth is shown in mesh with a rack.</p>	<p>The diagram shows a gear tooth profile with an addendum S_a and a dedendum S_b. The tooth is shown in mesh with a rack.</p>
Basic Principle	

Gear design is driven by manufacturing (cutting tool profile parameters).	Gear design is driven by application (performance parameters).
Application	
General Application Gears <ul style="list-style-type: none"> • Stock gears. • Gearboxes with interchangeable gear sets (like old machine tools). • Mechanical drive prototyping. • Low production machined gears. 	Custom Application Gears <ul style="list-style-type: none"> • Plastic and metal molded, powder metal, die cast, and forged gears. • High production machined gears. • Gears with special requirements and for extreme applications.

Table 4 presents an example of a direct design gear set in comparison with the “best” traditionally designed gear set based on a 25° pressure angle generating tool. The “best”, in this case, means well-balanced gears with minimum bending stresses and relatively high efficiency. As shown in the table, Direct Gear Design results in gears with about 30% lower maximum bending stress and a higher contact ratio, allowing for an increase in the center distance deviation. Also, the gear efficiency is increased from 97% to 98%, which means 33% less mechanical losses and heat generation, resulting in higher reliability and longer life.

Table 4

Shared Attributes:	Pinion		Gear	
	Number of teeth	11		57
Operating Pressure Angle	25°			
Diametral Pitch	20			
Center Distance	1.700"			
Face Width	.472"		.394"	
Pinion Torque	14 in-lb			
Gear Profiles	Traditional Design (AGMA 201.2)		Direct Gear Design®	
Performance Parameters	Pinion	Gear	Pinion	Gear
Max. Bending Stress, psi	8100	8600	5800 (-28%)	6000 (-30%)
Contact Ratio	1.25		1.40	
Max. Center Offset	+0.020"		+0.028"	
Gear Efficiency	97%		98%	

CONCLUSION

Direct Gear Design is an alternative approach to traditional gear design. It is not constrained by predefined tooling parameters. It allows for the analysis of a wide range of parameters for all possible gear combinations in order to find the most suitable solution for a particular custom application. This gear design method can exceed the limits of traditional rack generating methods of gear design.

Direct Gear Design results in a 15-30% reduction in stress level when compared to traditionally designed gears. This reduction can be translated into:

- Increased Load Capacity (15-30%)
- Size and Weight Reduction (10-20%)
- Longer Life
- Cost Reduction
- Increased Reliability
- Noise and Vibration reduction
- Increased Gear Efficiency
- Maintenance Cost Reduction

The Direct Gear Design[®] method has been implemented in a number of gear transmissions and other mechanisms where gears are used. In the automotive industry, this method has been applied for mechatronic actuators of drive-by-wire systems and supplemental mechanisms such as power locks, windows, mirrors, etc. There are still a lot of opportunities to maximize performance of automotive gearboxes, differentials, etc.

APPENDIX: Application examples

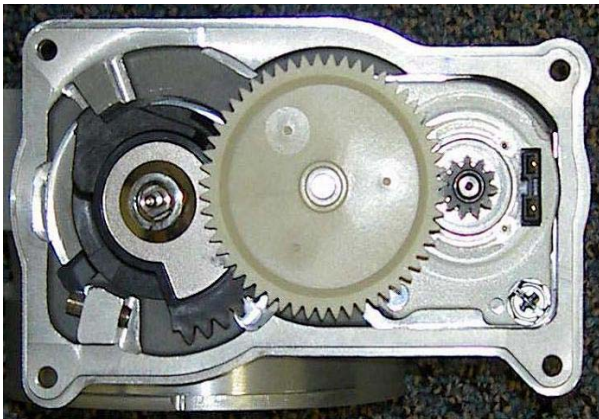
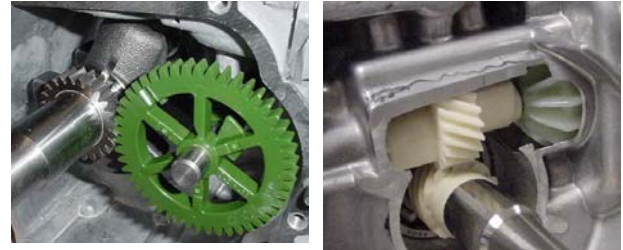


Fig. 18 Automotive gearbox

Fig. 18 presents an automotive gearbox that works in an under-the-hood environment with a temperature range of -48°C to $+135^{\circ}\text{C}$. Initially, it was designed with machined metal gears. Then plastic gears were considered for cost reduction. The thermal expansion of the plastic gears, the powder metal pinion, and the aluminum housing are very different from each other. The direct designed gears with long and flexible teeth

allowed for the absorption of all dimensional changes related to the variable operating conditions.



a
b
Fig. 19 The valve lifter gear drives

Fig. 19 shows spur and crossed helical valve lifter drives for a small piston engine. The gearbox operates at temperatures up to $+160^{\circ}\text{C}$. Direct Gear Design and proper plastics selection provided significant cost reduction.



Fig. 20 Planetary gearbox

The planetary drive for an automation application is shown in Fig. 20. The goal was to provide a high gear ratio (63:1) in one planetary stage and high output torque in a very limited space. The gear arrangement optimization in combination with Direct Gear Design allowed achieving this goal.



Fig. 21 Crossed helical gearbox

The crossed helical gears presented in Fig. 21 are used in agricultural equipment. The freedom of Direct Gear Design made it possible to replace an old style metal chain drive with a less expensive, safe, and maintenance free plastic gear drive and flexible shaft.



Fig. 22 Medical pump

Fig. 22 presents a gear pump. The direct designed flexible asymmetric teeth provide a better seal between the gears and the housing, resulting in higher output pressure, flow, and efficiency.

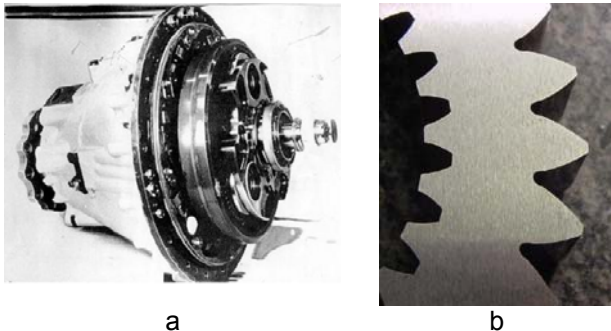


Fig. 23 The gearbox and sun gear of the turboprop engine

The two-stage planetary gearbox with asymmetric teeth for the turboprop engine and its sun gear are shown in the Fig. 23. This gear design solution delivered a significant increase in load capacity, reduced weight, and higher efficiency and reliability.

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