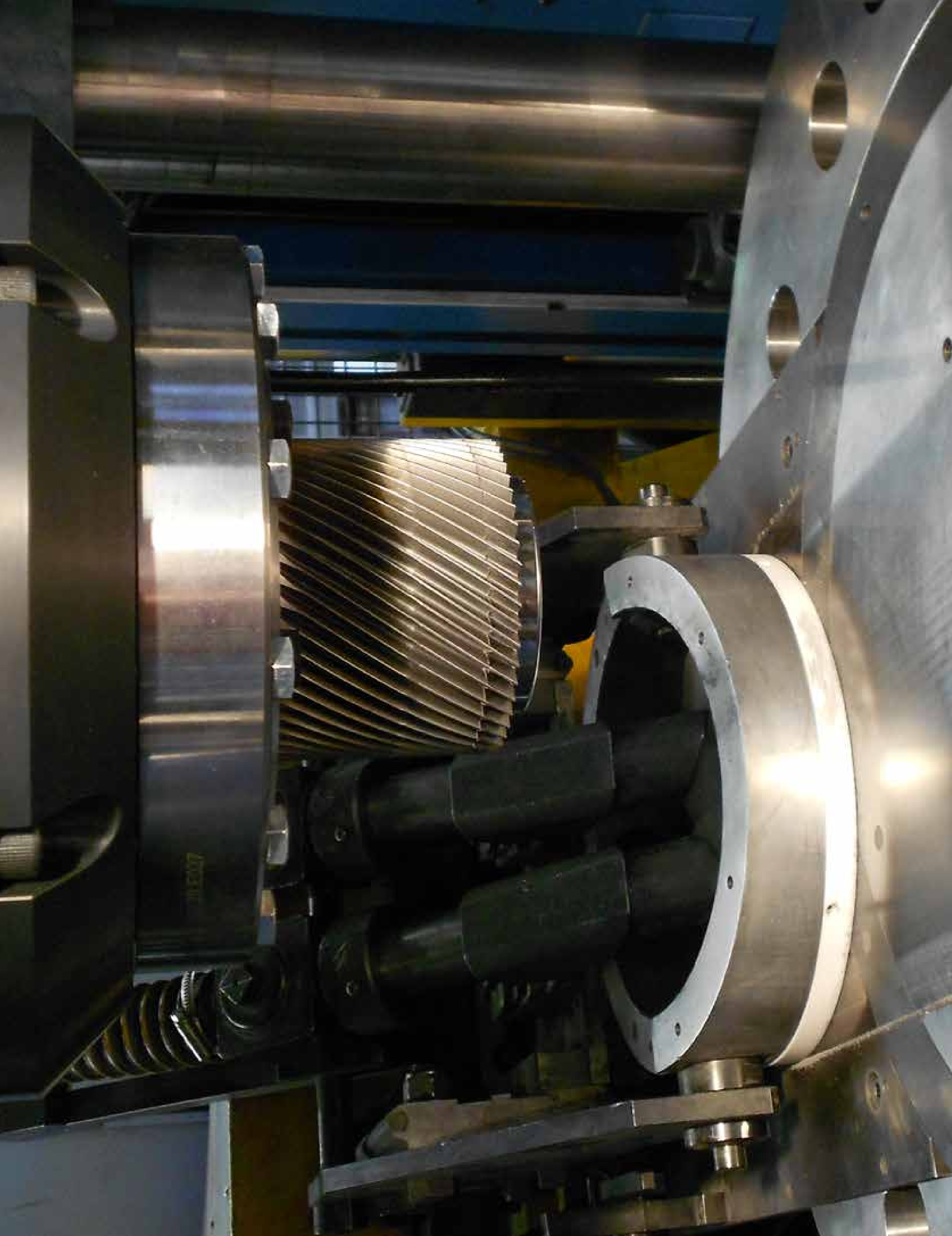

Contact ratio optimization of powder-metal gears

Price and the ability to offer a much larger design window are making manufacturers look to powder metal gears as a solution for high-performance gears.

By Alexander Kapelevich and Anders Flodin

POWDER METAL (PM) ALLOYS ARE BECOMING more of a solution for high-performance gears, not only because of part price but also because the technology offers a wider design window. Since powder metal alloys have a lower modulus of elasticity and Poisson ratio — the factors that amplify gear-tooth deflections — the design window opens up further.

The contact ratio is a critical gear-mesh parameter that greatly affects gear-drive performance, including load capacity, noise, and vibration. Gear-drive operating load produces bending and contact tooth deflections, which increase the actual effective contact ratio.

This article describes the analysis and gear macro-geometry optimization of powder-metal gears with transitional nominal contact ratio ε_α = 1.7-1.85. Under the operating load, the effective contact ratio of these gears is increased to $\varepsilon_{\alpha e} \geq 2.0$ creating greater load sharing, as well as providing stress and transmission error reduction.

PM gear technology has the inherent ability to reduce the weight and inertia of the gear wheel, thus reducing mass and energy losses.

When designing PM gears, special attention has to be paid to the use of the correct material properties, meaning the modulus of elasticity and Poisson's ratio. Designers also can improve weight and dynamics by understanding the possibilities that PM offers through its unique production methods. The PM process route also offers a direct reduction of the number of manufacturing steps, leading to improved cost performance.

The modulus of elasticity and Poisson's ratio can be empirically calculated as a function of density by equations 1 and 2 from [1].

$$E = E_0 \cdot \left(\frac{\rho}{\rho_0}\right)^{3.4}$$ **Equation 1**

$$\upsilon = \left(\frac{\rho}{\rho_0}\right)^{0.16} \cdot (1+\upsilon_0) - 1$$ **Equation 2**

EFFECTIVE CONTACT RATIO AND TRANSMISSION ERROR

In a spur gear mesh, the effective contact ratio can be defined as the ratio of the tooth engagement angle to the angular pitch. The tooth engagement angle is the gear's rotation angle from the start of the tooth engagement with the mating gear tooth to the end of the engagement. The effective contact ratio is [2]

$$\varepsilon_{\alpha e} = \frac{\varphi_1}{360/z_1} = \frac{\varphi_2}{360/z_2}$$ **Equation 3**

where:

ϕ_1 and ϕ_2 —pinion and gear engagement angles.

z_1 and z_2 — pinion and gear numbers of teeth.

$360/z_1$ and $360/z_2$ — pinion and gear angular pitches.

The gear mesh load is

$$F = 2000 T_1 / d_{bd1}$$ **Equation 4**

where T_1 — pinion operating torque in Nm, d_{bd1} — pinion drive flank base diameter in mm.

The load-sharing factor is

$$L = F_{\max} / F \times 100\%$$ **Equation 5**

where F_{max} — maximum contact load in the single tooth contact. If the effective contact ratio $\varepsilon_{\alpha de} \leq 2.0$, the load sharing factor L = 100%. If the effective contact ratio $\varepsilon_{\alpha de} > 2.0$, the load sharing factor L < 100%.

The transmission error is [3]

$$TE = r_{b2}(\theta_2 - u\theta_1)$$ **Equation 6**

where:

θ_1 and θ_2 — driving pinion and driven gear rotation angles.

r_{b2} — driven gear base radius.

A typical transmission error chart for a spur gear pair with the effective contact ratio $1.0 < \varepsilon_{\alpha e} < 2.0$ is shown in Fig. 1.

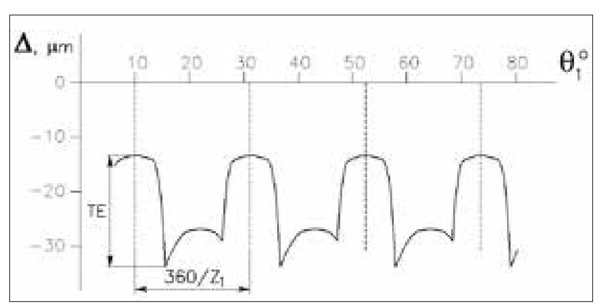

Fig. 1. Transmission error chart; Δ – distance in microns between actual tooth contact point and ideal contact point, which is defined ignoring manufacturing tolerances and operating conditions .

The effective contact ratio and transmission error are influenced by manufacturing tolerances, assembly

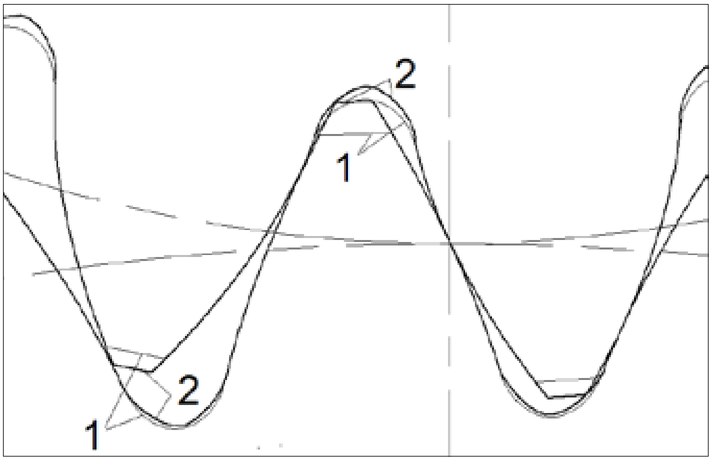

Fig. 2. Gear pair comparison overlay; 1 – (thin lines) traditionally designed gear tooth profiles, 2 – (thick lines) directly designed gear tooth profiles

misalignments, and operating conditions — including the gear's and other gearbox components' deflections under load, their thermal expansion or shrinkage, etc. In this article, only the bending and contact tooth deflections are considered for the definition of the effective contact ratio and transmission error. Each angular position of the driven gear relative to the driving gear is iteratively defined by equalizing the sum of the tooth contact load moments of each gear to its applied torque. The related tooth contact loads are also iteratively defined to conform to tooth bending and contact deflections, where the tooth bending deflections in each contact point are determined based on the FEA-calculated flexibility, and the total contact deflection is calculated based on the Hertz equation [4].

TRADITIONAL VS. DIRECT GEAR DESIGN

Table 1 and Figure 2 present comparison of the traditionally designed gear pair and the gear pair defined by the Direct Gear Design® method [5]. The traditionally designed gear pair has tooth macrogeometry used for high-performance gear transmissions. It is generated by the full tip radius tooling rack with a 25-degree pressure angle. The directly designed gear pair has the same pressure angle and the same tooth thicknesses at the pitch diameters as in the traditionally designed gears. The assumed average friction coefficient in both gear sets is equal to 0.05. Unlike in traditional gear design, Direct Gear Design allows selecting a desirable nominal contact ratio, setting its value as an input parameter. In this case, the nominal contact ratio is chosen to achieve an effective contact ratio under operating load that is equal to or slightly greater than 2.0. In addition, the directly designed gears have optimized tooth root fillets [5] to minimize bending stress concentration. The gear material is Höganäs Astaloy Mo 0.25%C with a modulus of elasticity of 160,000 MPa (density 7.27 g/cm3) and Poisson's ratio of 0.28. Because of the lower modulus of elasticity and Poisson's ratio, the traditionally and directly designed PM gears from Table 1 have bending and contact tooth deflections that are 27 percent to 30 percent greater than if they were made out of the case-harden gear steel with a modulus of elasticity of 206,000 MPa and Poisson's ratio of 0.3. As a result of combining the PM alloy properties with the gear tooth macrogeometry optimized by Direct Gear Design, the transitional nominal contact ratio gears, under the operating load, become the high effective contact ratio gears — with the load shared between two or three pairs of teeth, reducing bending and contact stresses and transmission error.

Gear Design Method		Traditional	Direct
Numbers of Teeth	pinion	23	23
	gear	46	46
Module, mm		4.348	4.348
Pressure Angle, °		25.0	25.0
Pitch Diameter (PD), mm	pinion	100.000	100.000
	gear	200.000	200.000
Tooth Tip Diameter, mm	pinion	109.74	110.85
	gear	207.66	209.94
Tooth Root Diameter, mm	pinion	90.19	89.27
	gear	188.12	188.28
Tooth Thickness at PD, mm	pinion	7.318	7.318
	gear	6.347	6.347
Face Width, mm	pinion	32.00	32.00
	gear	32.00	32.00
Center Distance, mm		150	150
Nominal Contact Ratio		1.45	1.74
Gear mesh efficiency, %		99.2	99.1(-0.1%)
Pinion Torque, Nm		2000	2000
Bending Stress, MPa	pinion	640	493(-23%)
	gear	608	472(-22%)
Contact Stress, MPa		1653	1526(-8%)
Max. Bending Deflection, µm	pinion	36.7	41.4
	gear	41.8	49.2
Max. Total Contact Deflection, µm		10.2	8.7
Effective Contact Ratio		1.79	2.03
Transmission Error, µm		27.9	17.9(-36%)

Table 1 Traditional and Direct Gear Design comparison

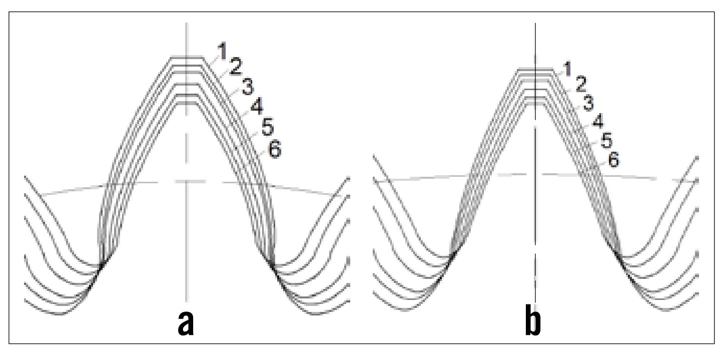

Fig. 3. Gear profiles overlay; a – pinion tooth profiles, b – gear tooth profiles

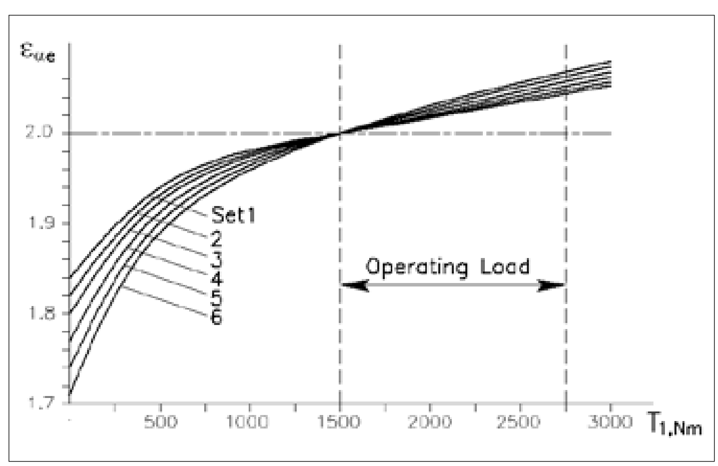

Fig. 4. Effective contact ratio vs. pinion torque

Gear Set Number		1	2	3
Numbers of Teeth	pinion	17	18	19
	gear	34	36	38
Module, mm		5.882	5.556	5.263
Pressure Angle, °		21.5	22.3	23.0
Pitch Diameter (PD), mm	pinion	100.000	100.000	100.000
	gear	200.000	200.000	200.000
Tooth Tip Diameter, mm	pinion	115.49	114.52	113.62
	gear	213.72	213.00	212.30
Tooth Root Diameter, mm	pinion	84.98	86.01	86.74
	gear	182.96	184.27	185.14
Tooth Thickness at PD, mm	pinion	9.942	9.355	8.920
	gear	8.537	8.098	7.614
Nominal Contact Ratio		1.84	1.82	1.80
Effective Contact Ratio at 1500 Nm Pinion Torque		2.00	2.00	2.00
Gear mesh efficiency, %		98.72	98.81	98.88

Table 2 Gear parameters

Gear Set Number		4	5	6
Numbers of Teeth	pinion	21	23	25
	gear	42	46	50
Module, mm		4.762	4.348	4.000
Pressure Angle, °		24.1	25.00	25.5
Pitch Diameter (PD), mm	pinion	100.000	100.000	100.000
	gear	200.000	200.000	200.000
Tooth Tip Diameter, mm	pinion	112.16	110.85	109.84
	gear	211.14	209.94	209.18
Tooth Root Diameter, mm	pinion	88.14	89.27	90.26
	gear	186.90	188.28	189.44
Tooth Thickness at PD, mm	pinion	7.999	7.318	6.624
	gear	6.961	6.347	5.943
Nominal Contact Ratio		1.77	1.74	1.71
Effective Contact Ratio at 1500 Nm Pinion Torque		2.00	2.00	2.00
Gear mesh efficiency, %		99.01	99.10	99.19

Table 3 Gear parameters (continued)

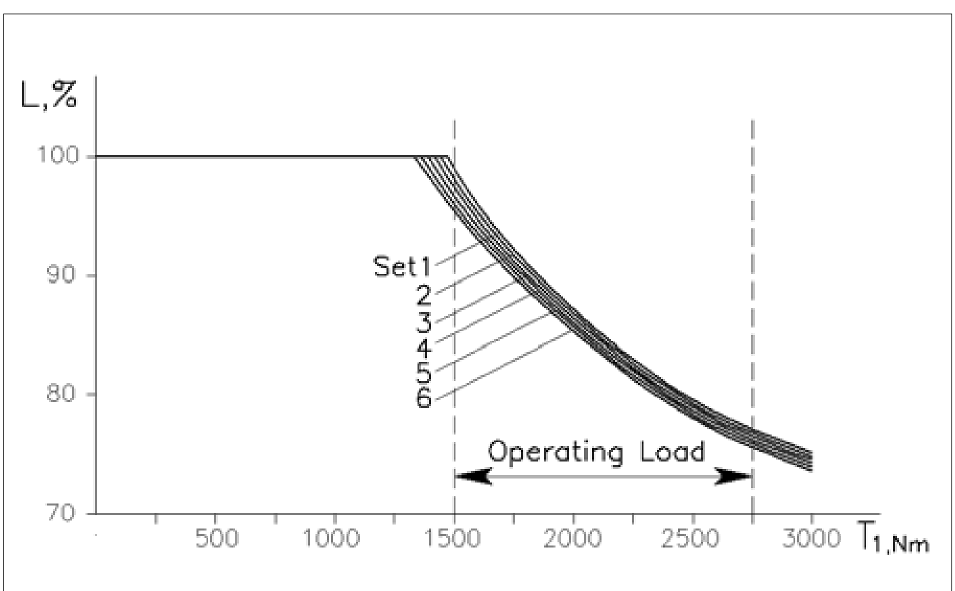

Fig. 5. Load sharing vs. pinion torque

Gear Set Number	1	2	3	4	5	6
Pinion Torque = 500 Nm						
Effective Contact Ratio	1.94	1.93	1.92	1.91	1.90	1.89
Load Sharing, %	100	100	100	100	100	100
Transmission Error, μm	9.2	8.9	8.6	7.9	7.2	6.5
Pinion Torque = 1000 Nm						
Effective Contact Ratio	1.98	1.98	1.97	1.97	1.96	1.96
Load Sharing, %	100	100	100	100	100	100
Transmission Error, μm	18.5	17.8	17.1	15.8	14.4	13.1
Pinion Torque = 1500 Nm						
Effective Contact Ratio	2.00	2.00	2.00	2.00	2.00	2.00
Load Sharing, %	99.0	98.1	97.4	96.6	96.0	95.4
Transmission Error, μm	23.0	22.5	22.0	20.9	19.8	18.6
Pinion Torque = 2000 Nm						
Effective Contact Ratio	2.02	2.02	2.02	2.03	2.03	2.03
Load Sharing, %	87.2	86.9	86.6	86.3	86.0	85.7
Transmission Error, μm	22.1	20.6	20.0	19.0	17.9	16.9
Pinion Torque = 2500 Nm						
Effective Contact Ratio	2.03	2.03	2.04	2.04	2.04	2.05
Load Sharing, %	79.6	79.3	79.0	78.7	78.4	78.1
Transmission Error, μm	19.7	19.2	18.7	17.6	16.5	15.5
Pinion Torque = 3000 Nm						
Effective Contact Ratio	2.05	2.06	2.06	2.07	2.07	2.08
Load Sharing, %	75.1	74.8	74.5	74.2	73.9	73.6
Transmission Error, μm	19.1	18.6	18.1	17.0	15.9	14.9

Table 4 Effective contact ratios, load sharing, and transmission errors.

Gear Set Number		1	2	3	4	5	6
Pinion Torque = 500 Nm							
Bending Stress, MPa	pinion	107	115	122	135	149	163
	gear	104	110	117	130	143	156
Contact Stress, MPa		1136	925	853	842	832	821
Pinon Torque = 1000 Nm							
Bending Stress, MPa	pinion	218	231	244	269	295	321
	gear	210	222	234	258	283	307
Contact Stress, MPa		1614	1304	1200	1185	1170	1155
Pinion Torque = 1500 Nm							
Bending Stress, MPa	pinion	295	315	333	372	402	434
	gear	286	319	342	355	385	415
Contact Stress, MPa		1981	1566	1426	1410	1395	1379
Pinion Torque = 2000 Nm							
Bending Stress, MPa	pinion	388	405	423	459	493	528
	gear	373	389	406	440	472	506
Contact Stress, MPa		2290	1743	1573	1550	1526	1503
Pinion Torque = 2500 Nm							
Bending Stress, MPa	pinion	486	502	521	556	591	626
	gear	466	482	500	533	566	599
Contact Stress, MPa		2562	1910	1715	1684	1652	1621
Pinion Torque = 3000 Nm							
Bending Stress, MPa	pinion	583	600	618	654	688	723
	gear	559	575	592	626	659	692
Contact Stress, MPa		2808	2070	1855	1815	1775	1735

Table 5 Bending and contact stresses.

COMPARABLE GEAR ANALYSIS

Tables 2 and 3 present gear parameters of six gear sets with different numbers of teeth. These gear sets are optimized by the Direct Gear Design method to satisfy the following conditions: center distance — 150 mm; gear ratio — 2:1; pinion and gear face widths — 32 mm; tooth tip thickness — about 0.30·module; effective contact ratio at the 1,500 Nm pinion torque is equal to 2.0; assumed average friction coefficient — 0.05; pinion and gear material is Höganäs Astaloy Mo 0.25%C with the modulus of elasticity – 160,000 MPa and Poisson's ratio — 0.28. Operating load range of all gear sets lays between 1,500 Nm and 2,500 Nm of driving pinion torque. All gears have optimized tooth root fillets.

Overlays of the pinion and gear tooth profiles of the analyzed six sets are shown in Figure 3. It indicates a significant difference in gear tooth sizes.

Tables 4 and 5 present the effective contact ratios, pinion and gear bending stress, contact stress, and transmission error of the analyzed six gear sets under various pinion torque values.

The effective contact ratio vs. the pinion torque chart is presented in Figure 4. When the pinion torque is zero, the effective contact ratio values are equal to the nominal contact ratio values because tooth deflections are zero. Increasing pinion torque T_1 increases the effective contact ratio that reaches its value of 2.0 at T_1 = 1,500 Nm. Further pinion torque growth provides load sharing between two or three pairs of teeth and an effective contact ratio $\varepsilon_{\alpha e} > 2.0$.

When $\varepsilon_{\alpha e} > 2.0$, the single tooth maximum load is reduced below 100 percent, typical for a conventional gear mesh that has $\varepsilon_{\alpha e} < 2.0$ (Figure 5). The higher the pinion's torque, the lower the single tooth maximum load. The tooth deflections become lower, respectively reducing the transmission error (TE). Figure 6 charts indicate that with $\varepsilon_{\alpha e} > 2.0$ the transmission error decreases, stays flat, and then gradually increases beyond the operating load range.

Figures 7 and 8 present the pinion and gear bending stress vs. pinion torque charts, which show the high stress increase gradient for low pinion torque values that produce an effective contact ratio $\varepsilon_{\alpha e} < 2.0$. When pinion torque values provide an $\varepsilon_{\alpha e} > 2.0$, the stress increase gradient is lower because the share of the maximum tooth load becomes lower. This allows for reduced bending stress in gears with transitional nominal contact ratio gears when compared to low nominal contact ratio gears. A similar effect of the transitional contact ratio is observed for the contact stress with growing pinion torque (Figure 9).

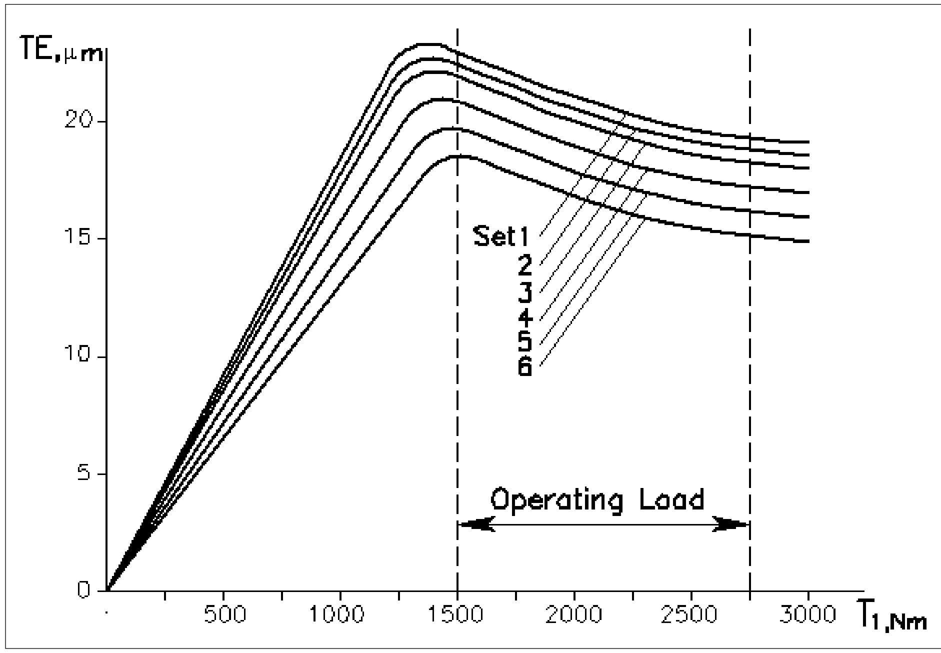

Fig. 6. Transmission vs. pinion torque

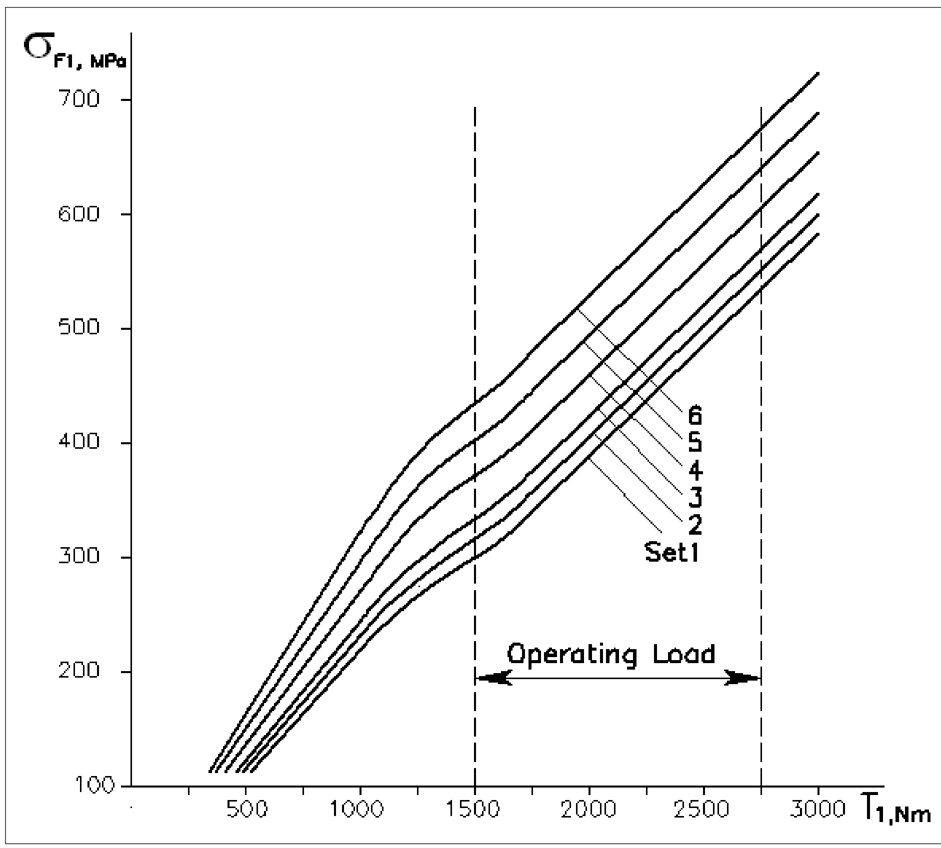

Fig. 7. Pinion bending stress vs. pinion torque

SUMMARY

This article describes the analysis of powder metal gears with a transitional nominal contact ratio ε_{α} = 1.7-1.85. A combination of the PM alloys' properties, the PM compaction technology, and tooth macro-geometry optimized by Direct Gear Design allows for increased bending and contacting tooth deflections. Under the operating load, these gears become high contact ratio gears with an effective contact ratio $\varepsilon_{\alpha e} \geq 2.0$, reducing bending and contact stresses and transmission error. Unlike solid steel gears, whose design is typically based on the rack generation process producing the trochoidal tooth, the PM gear compaction technology allows us to use an optimized root fillet profile, additionally reducing bending stress.

The article compares several gear sets with a transitional contact ratio and different numbers of teeth, while keeping the same gear ratio, center distance, gear face widths, and PM material. This comparison helps to select a suitable gear set depending on the gear drive performance priorities and limitations.

Results of this study might be useful for automotive transmissions, where application of the powder metal alloy gears is considered prospective.

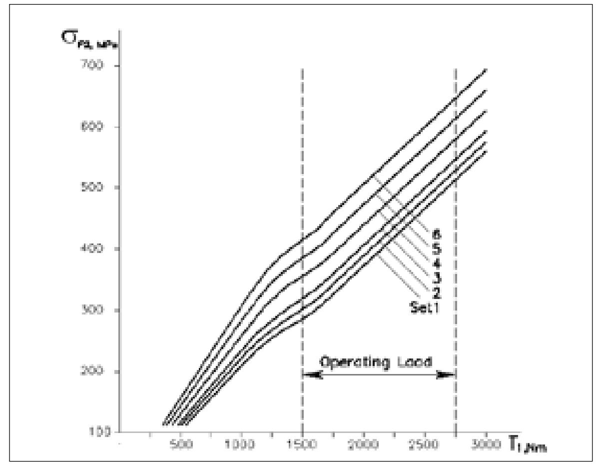

Fig. 8. Gear bending stress vs. pinion torque

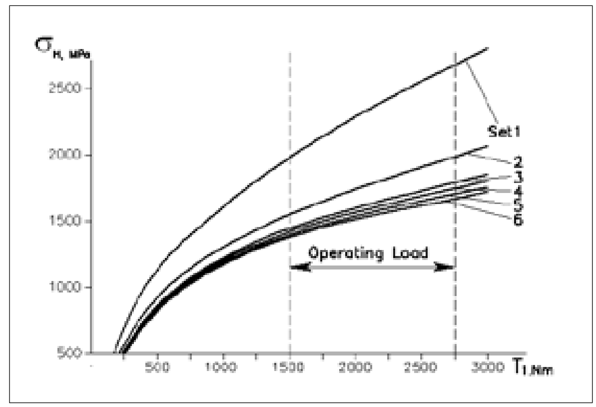

Fig. 9. Contact stress vs. pinion torque

REFERENCES

1. Flodin, A. Forden, L., Root and contact stress calculations in surface densified PM gears, Proceedings from World PM2004 Conference Vol.2. (2004), pp 395-400.
2. Kapelevich, A., Shekhtman, Y., Analysis and Optimization of Contact Ratio of Asymmetric Gears, Proceedings of the International Conference on Power Transmissions ICPT 2016-Chongqing, October 27-30, (2016).
3. D.P. Townsend, Dudley's Gear Handbook. 1967. 2nd edition. McGraw Hill. New York: 1040
4. Young, W.C., Budynas, R. G., Roark's Formulas for Stress and Strain, Seventh Edition, McGraw-Hill (2002).
5. Kapelevich, A., Direct Gear Design. CRC Press. (2013).

ABOUT THE AUTHORS: Alexander Kapelevich operates the gear-design consulting firm AKGears, LLC in Shoreview, Minnesota. He has a master's degree from Moscow Aviation Institute and a Ph.D from Moscow State Technical University and 40 years of experience with custom gear drive development. He is author of the book, "Direct Gear Design," and many technical articles.

Anders Flodin received his doctoral degree in 2000 from KTH in Stockholm on the topic wear modeling of tooth flanks of cylindrical gears. He is with Höganäs, Sweden, and is working with developing powder metal gear technology for automotive applications. Flodin has been involved with transmission development for helicopters, ships, and cars and has 15 years experience with PM gears.