Asymmetric Gears: Parameter Selection Approach

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Introduction

In many gear transmissions, a tooth load on one flank is significantly higher and is applied for longer periods of time than for the opposite one; an asymmetric tooth shape reflects this functional difference.

There are publications addressing gears with asymmetric teeth (Refs. 1–2) where the tooth geometry is defined by the pre-selected asymmetric generating (tooling) gear rack parameters. A similar approach is commonly used in traditional gear design of conventional gears with symmetric teeth. With asymmetric gears, the standard symmetric tooling gear rack is modified by altering the pressure angle of one of its flanks. However, such a simplified approach to asymmetric gear design greatly limits opportunities to maximize performance for a wide variety of possible applications for these gears.

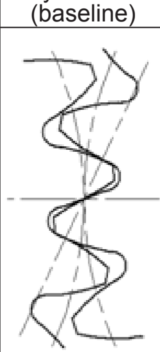
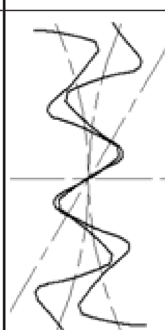
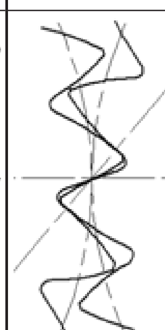
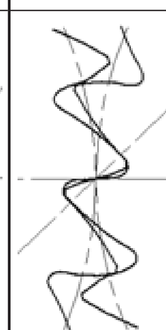
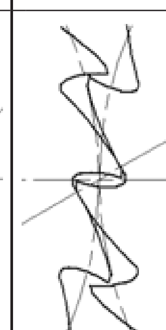
The alternative Direct Gear Design method is not bound by the preselected basic rack parameters and provides asymmetric gear tooth geometry optimized for specific gear drive applications (Refs. 3–4).

This paper describes an approach that rationalizes the degree of asymmetry (or asymmetry factor *K*) selection to meet a variety of operating conditions and requirements for custom gear drives.

Torque Transmission Conditions

Table 1 presents different torque transmission examples of spur gear pairs with identical, 24-tooth mating gears to illustrate bi-directional and unidirectional drive applications.

Table 1—Torque transmission examples of spur gear pairs with identical, 24-tooth mating gears

Case #	1	2	3	4	5
Load transmission	Bidirectional		Mostly unidirectional	Unidirectional	
Loaded flanks	both	both	drive, coast with lower load	drive, coast with no load	drive flank only
Tooth profile	Symmetric (baseline)	Symmetric	Asymmetric	Asymmetric	Asymmetric
Gear mesh					
Pressure angle, °	25	32	40/24*	46/10*	60/-**
Asymmetry coefficient	1.0	1.0	1.19	1.42*	-**
Contact ratio	1.35	1.2	1.2/1.44*	1.2/1.0	1.2/-**
Hertz contact stress, %	100	92	88/102*	86/150*	94/-**
Bearing load, %	100	107	118/99*	130/92*	181/-**
Specific sliding velocity, %	100	94	75/108*	68/97*	49/-**

* for drive/coast tooth flanks;
** coast flank mesh does not exist.

Examples 1 and 2. The gear teeth are symmetric and their surface durability is identical for both tooth flanks. Example 1 presents the traditionally designed, 25º pressure angle gears with full radius fillet. This example is considered as a baseline and its Hertzian contact stress, bearing load and specific sliding velocity are assumed as 100% for comparison with other gear examples. This type of gear profile is used in the aerospace industry because it provides better bending strength and flank surface endurance in comparison with the standard, 20º pressure angle gears typical for commercial applications. Example 2 uses high 32º pressure angle symmetric gears, optimized by the Direct Gear Design method. Their Hertzian contact stress is about 8% lower and the specific sliding velocity is about 6% lower than for the baseline gear pair. This should provide better flank tooth surface pitting or scoring resistance. However, the bearing load is 7% higher.

Example 3.These asymmetric gears are for mostly unidirectional load transmission with a 40º pressure angle driving tooth flanks providing 12% contact stress and 25% sliding velocity reduction. At the same time, the contact stress and sliding velocity of the coast flanks are close to these parameters of the baseline gears and should provide the tooth surface load capacity similar to the baseline gears. These types of gears may find applications for drives with one main load transmission direction, but they should be capable of carrying a lighter load for shorter periods of time in the opposite direction.

Example 4. These asymmetric gears have a 46º drive pressure angle that allows a reduction in contact stress by 14% and sliding velocity by 32%. A disadvantage of such gear teeth is a very high, i.e., + 30%, bearing load. These types of gears are only for unidirectional load transmission. Their 10º coast pressure angle flanks have insignificant load capacity. These types of gears may find applications for drives with only one load transmission direction that may occasionally have no-load coast flank tooth contact, as in the example of a tooth bouncing in high-speed transmissions.

Example 5. Asymmetric gears have only driving tooth flanks with the extreme 60º pressure angle and with no involute coast tooth flanks at all. As a result the bearing load is significant.

Asymmetry Factor Selection

Gear Pair

The gear asymmetry factor K is:

$$K = \frac{\cos \alpha_{wc}}{\cos \alpha_{wd}} \quad (1)$$

where:

α_{wd} = drive pressure angle;

α_{wc} = coast pressure angle

There are many applications—e.g., Example 3—where a gear pair transmits load in both load directions, but with significantly different magnitude and duration (Fig. 1). In this example the gear asymmetry factor K can be defined by equalizing the potential accumulated tooth surface damage that depends on operating contact stress and the number of the tooth flank load cycles. In other words, the contact stress safety factor S_H should be the same for both the drive and coast tooth flanks. This condition can be presented as:

$$S_H = \frac{\sigma_{HPd}}{\sigma_{Hd}} = \frac{\sigma_{HPc}}{\sigma_{Hc}} \quad (2)$$

where:

σ_{Hd} and σ_{Hc} = operating contact stresses for the drive and coast tooth flanks

σ_{HPd} and σ_{HPc} = permissible contact stresses for the drive and coast tooth flanks that depend on the number of load cycles

Then from Equation 2:

$$\frac{\sigma_{Hd}}{\sigma_{Hc}} = \frac{\sigma_{HPd}}{\sigma_{HPc}} \quad (3)$$

The contact stress at the pitch point (Ref. 5) is:

$$\sigma_H = Z_H Z_E Z_\varepsilon Z_\beta \sqrt{\frac{F_t}{d_{w1} b_w} \frac{u \pm 1}{u}} \quad (4)$$

where:

$$Z_H = \sqrt{\frac{2\cos(\beta_b)\cos(\alpha_{wt})}{\cos(\alpha_t)\sin(\alpha_{wt})}} = \text{zone factor that for the directly designed spur gears is} \quad (5)$$

$$Z_H = \frac{2}{\sqrt{\sin(\alpha_w)}}$$

Z_E = Elasticity factor that takes into account gear material properties (modulus of elasticity and Poisson's ratio)

Z_ε = Contact ratio factor; its conservative value for spur gears is $Z_\varepsilon = 1.0$

Z_β = Helix factor, for spur gears $Z_\beta = 1.0$

F_t = Nominal tangent load, that at the pitch diameter d_{w1} is

$$F_t = \frac{2T_1}{d_{w1}}$$

T_1 = Pinion torque

b_w = Contact face width

sign "+" for external gearing; sign "-" for internal gearing.

For the directly designed spur gears, the contact stress at the pitch point can be presented as

$$\sigma_H = Z_E \frac{2}{d_{w1}} \sqrt{\frac{2T_1}{b_w \sin(2\alpha_w)} \frac{u \pm 1}{u}} \quad (6)$$

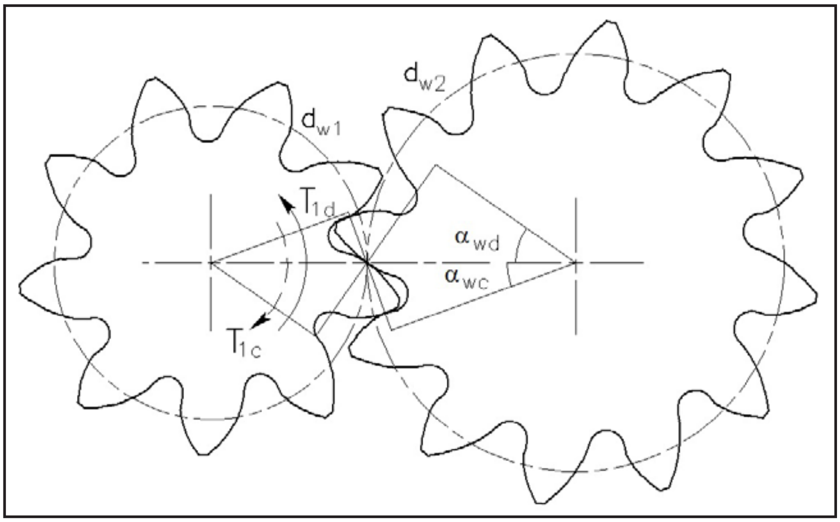


Figure 1—Asymmetric gear pair.

Now this equation can be presented for the drive and coast flank contact, and be used for Equation 3:

$$\frac{\sin(2\alpha_{wc})}{\sin(2\alpha_{wd})} = A \quad (7)$$

where:

$$A = \frac{T_{1c}}{T_{1d}}\left(\frac{\sigma_{HPd}}{\sigma_{HPc}}\right)^2 \quad (8)$$

= coefficient that reflects a difference in the applied load and number of cycles for the drive and coast tooth flanks

T_{1d} and T_{1c} = pinion torque applied to the drive and coast tooth flanks

According to Reference 5, "The permissible stress at limited service life or the safety factor in the limited life stress range is determined using life factor Z_{NT}." This allows substitution of the permissible contact stresses in Equation 8 for the life factors:

$$A = \frac{T_{1c}}{T_{1d}}\left(\frac{Z_{NTd}}{Z_{NTc}}\right)^2 \quad (9)$$

When the coefficient A is defined and the drive pressure angle selected, the coast pressure angle and asymmetry coefficient are calculated by Equations 7 and 1, accordingly.

If the gear tooth is equally loaded in both main and reversed rotation directions, both the coefficient A and the asymmetry factor K are equal to 1.0 and the gear teeth are symmetric.

Example 1: The drive pinion torque T_{1d} is two times greater than the coast pinion torque T_{1c}. The drive tooth flank has 10^9 load cycles and the coast tooth flank has 10^6 load cycles during gear drive life. From the *S-N* curve (Ref. 5) for steel gears, an approximate ratio of the life factors $Z_{NTd}/Z_{NTc}=0.85$. The coefficient $A=0.85^2/2=0.36$. Assuming the drive pressure angle is $\alpha_{wd}=36°$, the coast pressure angle from Equation 7 is $\alpha_{wc}=10°$ and the asymmetry factor from Equation 1 is $K=1.22$.

In many unidirectional gear drives such as, for example, where propulsion system transmissions may seem irreversible, the coast tooth flanks are loaded due to the system inertia during the drive system deceleration or the tooth bouncing in the high-RPM drives. This can be significant and should be taken into consideration while defining the asymmetry factor K.

If the gear drive is completely irreversible and the coast tooth flanks never transmit any load (Examples 4 and 5), the asymmetry factor is defined exclusively by the drive flank geometry. In this example an increase in the drive flank pressure could be limited by a minimum selected contact ratio and by separating load applied to the bearings. Application of a very high drive flank pressure angle results in reduced coast flank pressure angle and, possibly, an involute profile undercut near the tooth root. Another limitation of the asymmetry factor of the irreversible gear drive is a growing, compressive bending stress at the coast flank root. As is typical for conventional symmetric gears, compressive bending stress does not present a problem because its allowable limit is significantly higher than that of tensile bending stress. However, for asymmetric gears it may become an issue—especially for gears with thin rims.

Gears in Chain Arrangement

In the chain gears, the idler gear transmits the same load by both tooth flanks. While this arrangement may seem unsuitable for asymmetric gear application, in many examples the idler's mating gears have a significantly different number of teeth (Fig. 2). This allows application of asymmetric gears to equalize the contact stress and to achieve maximum load capacity.

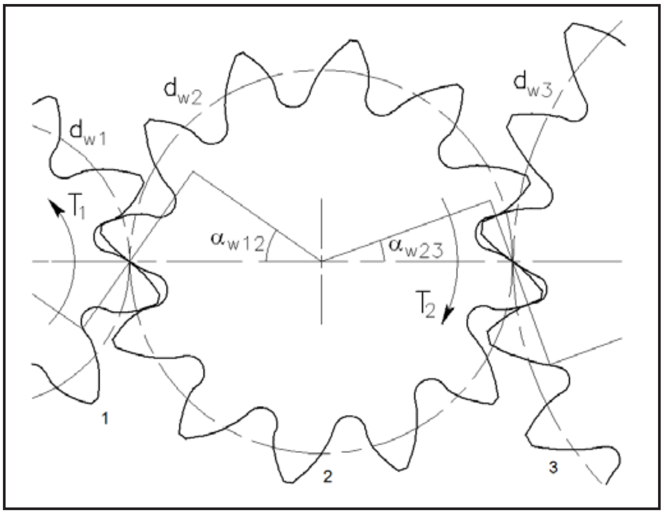


Figure 2—Chain gear arrangement: 1 = input pinion; 2 = idler gear; 3 = output gear.

Equation 6 is used to define the pitch point contact stress in the pinion/idler gear mesh:

$$\sigma_{H12} = Z_E \frac{2}{d_{w1}}\sqrt{\frac{2T_1}{b_{w12}\sin(2\alpha_{w12})}\,\frac{u_{12}+1}{u_{12}}} \quad (10)$$

and in the idler/output gear mesh:

$$\sigma_{H23} = Z_E \frac{2}{d_{w2}}\sqrt{\frac{2T_2}{b_{w23}\sin(2\alpha_{w23})}\,\frac{u_{23}+1}{u_{23}}} \quad (11)$$

or, ignoring gear mesh losses:

$$\sigma_{H23} = Z_E \frac{2}{u_{12}d_{w1}}\sqrt{\frac{2u_{12}2T_1}{b_{w23}\sin(2\alpha_{w23})}\,\frac{u_{23}+1}{u_{23}}} \quad (12)$$

where the subscript indexes "12" and "23" are for the pinion/idler gear and the idler/output gear meshes, accordingly.

If all three chain gears are made from the same material, a condition $\sigma_{H12}=\sigma_{H23}$ describes the equal potential for accumulated tooth surface damage of the idler gear flanks. Using Equations 11 and 12, this condition can be presented as:

$$\frac{\sin(2\alpha_{23})}{\sin(2\alpha_{12})} = \frac{b_{w12}}{b_{w23}}\,\frac{u_{23}+1}{u_{23}(u_{12}+1)} \quad (13)$$

where:

$u_{12}=n_2/n_1$ and $u_{23}=n_3/n_2$ are the gear ratios

b_{w12} and b_{w23} are the contact face widths in the pinion/idler gear and the idler/output gear meshes, accordingly;

n_1, n_2 and n_3 are number of teeth of the input pinion, idler gear and output gear

When these parameters are known and the drive pressure angle in the pinion/idler gear mesh is selected, the coast pressure angle in this mesh and asymmetry coefficient are calculated by Equations 13 and 1, accordingly.

Example 2: The pinion number of teeth is $n_1 = 9$; the idler gear number of teeth is $n_2 = 12$; the output gear number of teeth is $n_3 = 20$; the contact face width ratio is $b_{w12}/\ b_{w23} = 1.2$; and the pinion/idler gear pressure angle is $\alpha_{w12} = 35^o$. Then the idler/output pressure angle from Equation 13 is $\alpha_{w23} = 25.3^o$ and the asymmetry factor K from Equation 1 is 1.104.

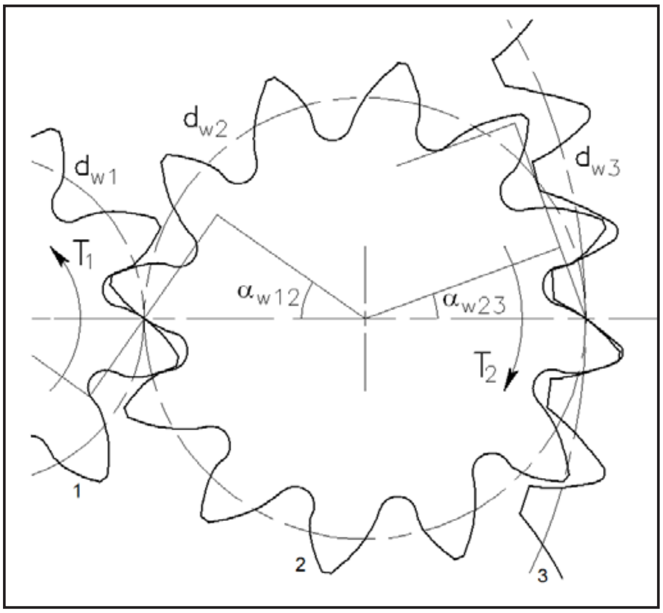


Figure 3—Planetary gear arrangement: 1 = sun gear; 2 = planet gear; 3 = ring gear.

Planetary Arrangement

A similar contact stress equalization technique can also be applied for the planetary gear arrangement (Fig. 3) because the planet gear is considered as the idler gear, engaged with the sun gear and ring gear.

In this example, considering that the planet and ring gears are in the internal mesh, Equation 13 looks like:

$$\frac{\sin(2\alpha_{w23})}{\sin(2\alpha_{w12})} = \frac{b_{w12}}{b_{w23}} \frac{u_{23} - 1}{u_{23}\,(u_{12} + 1)} \tag{14}$$

where:

$u_{12} = n_2/n_1$ and $u_{23} = n_3/n_2$ are the gear ratios

b_{w12} and b_{w23} are the contact face widths in the sun/planet gear and the planet/ring gear meshes, accordingly

n_1, n_2, and n_3 are number of teeth of the sun, planet and ring gears

When these parameters are known and the drive pressure angle in the sun/planet gear mesh is selected, the coast pressure angle in this mesh and asymmetry coefficient are calculated by Equations 14 and 1, accordingly.

Example 3: The sun gear number of teeth is $n_1 = 9$; the planet gear number of teeth is $n_2 = 12$; the output gear number of teeth is $n_3 = 33$; the contact face width ratio is $b_{w12}/\ b_{w23} = 1.8$; and the pinion/idler gear pressure angle is $\alpha_{w12} = 40^o$.

Then the planet/ring pressure angle from Equation 14 is $\alpha_{w23} = 14.5^o$ and the asymmetry factor K from Equation 1 is 1.264.

Summary

Selection of the gear tooth asymmetry factor K should be considered, depending on the gear drive application.

For an asymmetric gear pair that has different load application conditions in opposite directions, selection of the asymmetry factor K is based on equalizing of potential accumulated tooth surface damage in both load transmission directions.

For unidirectional chain and planetary gear arrangements, selection of the asymmetry factor K is based on equalizing of potential accumulated tooth surface damage in both flanks of idler (or planet) gear.

References

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Alex Kapelevich *possesses more than 30 years of custom gear research and design experience, as well as over 100 successfully accomplished projects for a variety of gear applications. His company, AKGears, provides consulting services—from complete gear train design (for customers without sufficient gear expertise) to retouching (typically tooth and fillet profile optimization) of existing customers' designs—in the following specific areas: traditional or direct gear design; current design refinement; R&D; failure and testing analysis. The company provides gear drive design optimization for increased load capacity; size and weight reduction; noise and vibration reduction; higher gear efficiency; backlash minimization; increased lifetime; higher reliability; cost reduction; and gear ratio modification and adjustment. Kapelevich is the author of numerous technical publications and patents, and is a member of the AGMA Aerospace and Plastic Gearing Committees, SME, ASME and SAE International. He holds a Ph.D. in mechanical engineering from Moscow State Technical University and a Masters Degree in mechanical engineering from the Moscow Aviation Institute.*